Dynamics: Nov. 11
Fig. 7.1  Natural coordinates $(s, n)$ defined at point P in a horizontal wind field. Curved arrows represent streamlines.

<table>
<thead>
<tr>
<th></th>
<th>Vectorial</th>
<th>Natural coords.</th>
<th>Cartesian coords.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$\frac{\partial V}{\partial n}$</td>
<td></td>
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<tr>
<td>Curvature</td>
<td>$V \frac{\partial \psi}{\partial s}$</td>
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<tr>
<td>Diffuence</td>
<td>$V \frac{\partial \psi}{\partial n}$</td>
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<tr>
<td>Stretching</td>
<td>$\frac{\partial V}{\partial s}$</td>
<td></td>
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<tr>
<td>Vorticity $\zeta$</td>
<td>$k \cdot \nabla \times \nabla$</td>
<td>$V \frac{\partial \psi}{\partial s} - \frac{\partial V}{\partial n}$</td>
<td>$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$</td>
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<tr>
<td>Divergence Div$\nabla V$</td>
<td>$\nabla \cdot \nabla$</td>
<td>$V \frac{\partial \psi}{\partial n} - \frac{\partial V}{\partial s}$</td>
<td>$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$</td>
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<tr>
<td>Deformation</td>
<td>$-V \frac{\partial \psi}{\partial n} - \frac{\partial V}{\partial s}$</td>
<td>$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$</td>
<td></td>
</tr>
</tbody>
</table>

* The signs of properties relating to vorticity are defined as positive for the northern hemisphere.
Which are non-divergent, irrational??
Circulation:

\[ C = \oint V_x ds = \iint \xi dA = \iint \text{Div}_H \mathbf{v} dA \]

Circulations often conserved.

**Deformation:**

\[ \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \]

⇒ Flow can be broken down into divergent, rotational, and deformation components

Orthonormal to div, vort.

Flow over mountains
Both deformation and shear strengthen frontal zones.
Effective gravity $g$, includes centrifugal force

Motion introduces additional force: Coriolis

$$ f = 2\Omega \sin \phi $$

$$ \Omega = 2\pi \text{ rad day}^{-1} = 7.292 \times 10^{-5} \text{ s}^{-1} $$

Flow to right of motion $\Rightarrow$ CW in NH
INERTIAL CIRCLES: less common in atmosphere (less inertia)

2 examples:
• land-sea breeze (Hsu, 1970, MWR)
• mountain slope/valley circulation (Rocky Mountains/Plains)
• katabatic flow?
Motion determined by balance between pressure gradient force, Coriolis force, frictional force

Midlatitude horizontal velocities $\sim 10$ m/s
$Dv/dt \sim 10^{-4}$ m s$^{-2}$
Coriolis force $\sim fV \sim 10^{-3}$ m s$^{-2}$

Geostrophic wind = balanced pressure/coriolis flow

$F_{x,y} =$ vertical gradient in horizontal momentum

$F = \frac{\partial \tau}{\partial z} \ast 1/\rho$
In Boundary Layer, frictional drag in evidence

Wind speed sub-geostrophic, drifts towards lower pressure

\[ fV = |P| \cos \psi \]
Gradient Wind: in regions of high curvature, add the centripetal acceleration to eqn of motion

Sub-geostrophic  Super-geostrophic
Thermal Wind: The vertical shear in geostrophic wind is related to the horizontal temperature gradient.

\[
(V_\phi)_2 - (V_\phi)_1 = \frac{1}{f}k \times \nabla(\Phi_2 - \Phi_1) \\
(V_\phi)_2 - (V_\phi)_1 = -\left(\frac{R\ln(p_1/p_2)}{p_2}\right)k \times \nabla(T)
\]

Barotropic atm $\Rightarrow$ grad $T = 0$ $\Rightarrow$ geostrophic wind constant

Equivalent barotropic
Again...

Explains why westerlies strongest At top of troposphere
Do these plots make sense??

30-yr Mean meridional Profile Of temp. and zonal Wind, from Australia to Eastern Siberia, January
Cold advection

Warm advection