Nov 10, class #21 MPO 551. Continuing on from last time.

RTE. 3º radiative parameter: for air per medium: \( \tau, \tilde{\omega}, j \)

\[ \begin{align*}
\text{start w}1 (0): & \quad \frac{dI(\tau)}{d\tau} = I(\tau) \\
& \quad \beta(x,y,\tau) \times \beta(x,\tau) j
\end{align*} \]

\( \tau = \int \beta(x,\tau) d\tau \)

so that \( \tau = \frac{e^\tau}{\mu} \).

for an exponential atmosphere: (fig 4.23 W1) . helpful for understanding IR spectra.

\( p(x) = p_o e^{-x/\mu} \)

\( p_o \) air density at sea level.
\( \mu \) for well-mixed gas (\( CO_2 \)).

\( p_i(x) = w_i p_o e^{-x/\mu} \) mixing ratio

\[ \begin{align*}
\beta &= k_e \beta_i = k_e w_i p_o e^{-2/\mu} \\
\tau &= \int \beta d\tau = k_w p_o H e^{-2/\mu} = k_w p_o H \\
T &= e^{-\tau/H} = \exp \left[ \frac{-k_w p_o H}{\mu} e^{-2/\mu} \right] \\
\text{Mean weighting function,}
\end{align*} \]
absorption: \( \Delta T \) ignoring scattering,
transmission at temperature \( \tau \)

\[
\text{abs}(z) = T(z_2) - T(z_1) = \left[ \frac{dT}{dz} = W(z) \right]
\]

\( \Rightarrow \) absorption depends on gradient in transmission.
\( \Rightarrow \) same for emission.

\[
W(z) = \frac{d}{dz} \left( e^{-\tau(z)} \right) \quad \text{w}I = \beta \frac{d\tau}{dz} = \beta
\]

\[
= \frac{\beta}{\mu} T \quad \text{(no assumptions about profile)}
\]

\( T = 0 \) (opaque) \( \Rightarrow \) no abs. except at very top.

\( T = 1 \) then \( \frac{dT}{dz} = 0 \) \( \Rightarrow \) no absorption.

for an exponential profile, \( \frac{dW}{dz} \) gives upper altitude of maximum absorption/emission.

works out to be \( \frac{\tau(z)}{\mu} = 1 \) (p. 190-193 Pettig)

sec 4.5 WH.

that's it for transmission, sec 4.5.3.

now include absorption/emission more explicitly:

\[
\frac{dT}{dz} = I - (1-W)B.
\]

\( \text{integrated form:} \ I(0) = I(e) e^{-\tau} + \int_0^e B e^{-\tau} d\tau \)

can be derived from \( dI = dI_{abs} + dI_{emit} = B (B-I) dz \)

can be rearranged, integrated form as.

\[
I(0) = I(e) e^{-\tau} + \int B d\tau \frac{dT}{dz} \Rightarrow I(0) = I(e) e^{-\tau} + \int B W(z) d\tau.
\]

from space:

\[
I'(0) = I'(e) T_{abs} + \int B(z) W(z) d\tau \approx I'(0) T_{abs} + BT_{abs}\left(1-T\right)
\]

\( T_{abs} \) = M-1 weighting function