Homework 1 solutions, MPO663, Feb 20 2006

I. Moist thermo
1. The static energy (linear, convenient) and entropy (invertible, exact) descriptions of thermodynamic profiles in this mean sounding are qualitatively very similar (top plots).

2. The exact parcel temperature profile (moist adiabat) is a few K warmer than the environmental temperature profile. Virtual temperature effects (the light molecular weight of water vapor) are concentrated at low levels, where the parcel has the most water vapor. The virtual effect reaches about 2-3K, but both parcel and environment have some vapor so its effect in terms of parcel buoyancy is less than this maximum magnitude. Condensate loading for a reversible parcel has a similar magnitude (a few K), but is concentrated at upper levels (where the parcel has the most condensed water). Comparing the crude parcel-environment T difference measure one would get from lifting a parcel with MSE conserved (heavy line on lower right), we see that its sign is reasonable but the profile of buoyancy is imperfect.
II. Having examined buoyancy, look at its countravailing force—the pressure gradient that quickly develops to resist the flow divergence that the buoyancy force alone would tend to create. To satisfy mass continuity, a rising parcel must push air out of the way and draw in air behind it. The pressure field is the agent of this mass continuity mandate. For our uniform B field the solution looks like this. The pressure field is dominated by a high above the parcel and low below, but has substructure associated with the turrets.
The solution is linear, so the turret effects can be isolated as shown below. Notice that the absolute magnitude of $p^*$ is scale dependent. This is because the $p^*$ field exists to counter the divergent component buoyancy force with a pressure gradient force, and the same pressure gradient can be achieved with less pressure perturbation when the pattern is of smaller spatial scale.
Here is the total vertical force per unit mass felt by the air, $B - \frac{dp}{dz}$. The parcel feels positive (upward) net force, so it would ascend. The turrets have especially strong upward force, while the air between the big turrets is actually forced downward, like the air on the sides of the “cumulus cloud”. Thus, the protrusions would tend to grow with time. This explains why buoyant cumuliform clouds are bumpy in shape.