Multiscale Convective Wave Disturbances in the Tropics: Insights from a Two-Dimensional Cloud-Resolving Model

STEFAN N. TULICH
CIRES and NOAA/Earth System Research Laboratory, Boulder, Colorado

BRIAN E. MAPES
RSMAS, University of Miami, Miami, Florida

(Manuscript received 20 November 2006, in final form 30 April 2007)

ABSTRACT

Multiscale convective wave disturbances with structures broadly resembling observed tropical waves are found to emerge spontaneously in a nonrotating, two-dimensional cloud model forced by uniform cooling. To articulate the dynamics of these waves, model outputs are objectively analyzed in a discrete truncated space consisting of three cloud types (shallow convective, deep convective, and stratiform) and three dynamical vertical wavelength bands. Model experiments confirm that diabatic processes in deep convective and stratiform regions are essential to the formation of multiscale convective wave patterns. Specifically, upper-level heating (together with low-level cooling) serves to preferentially excite discrete horizontally propagating wave packets with roughly a full-wavelength structure in troposphere and “dry” phase speeds \( c_w \) in the range 16–18 m s\(^{-1}\). These wave packets enhance the triggering of new deep convective cloud systems, via low-level destabilization. The new convection in turn causes additional heating over cooling, through delayed development of high-based deep convective cells with persistent stratiform anvils. This delayed forcing leads to an intensification and then widening of the low-level cold phases of wave packets as they move through convection regions. Additional widening occurs when slower-moving (\(-8\) m s\(^{-1}\) “gust front” wave packets excited by cooling just above the boundary layer trigger additional deep convection in the vicinity of earlier convection. Shallow convection, meanwhile, provides positive forcing that reduces convective wave speeds and destroys relatively small-amplitude-sized waves. Experiments with prescribed modal wind damping establish the critical role of short vertical wavelengths in setting the equivalent depth of the waves. However, damping of deep vertical wavelengths prevents the clustering of mesoscale convective wave disturbances into larger-scale envelopes, so these circulations are important as well.

1. Introduction

Tropical deep convection is often found to be organized into multiscale wave disturbances moving parallel to the equator. Satellite observations in Fig. 1, for example, reveal numerous large-scale \([O(1000 \text{ km})]\), zonally propagating “envelopes” of convection, within which are embedded numerous smaller-scale \([O(100 \text{ km})]\) disturbances moving eastward and westward. Such a hierarchical pattern of convective wave organization was documented early on by Nakazawa (1988), who referred to the large-scale, eastward-moving envelopes as “superclusters” (see also Sui and Lau 1992; Chen et al. 1996). More recent studies have shown that Nakazawa’s superclusters are, in fact, convectively coupled Kelvin waves with the dispersion properties of shallow-water Kelvin modes (Wheeler and Kiladis 1999; Straub and Kiladis 2002). The smaller-scale disturbances, meanwhile, are thought to be associated with higher-frequency inertia–gravity waves (Takayabu et al. 1996; Haertel and Johnson 1998).

Although our ability to detect and classify tropical wave hierarchies has improved over the years, we still do not have a clear understanding of their origin and dynamics. For example: do the large-scale wave envelopes in Fig. 1 merely act to modulate the occurrence and/or amplitude of the smaller-scale wave distur-
bances, or do interactions on small scales somehow lead to the growth of wave energy on larger scales? What determines the characteristic sizes and propagation speeds of the wave disturbances? How do all these waves, short and long, relate to individual convective cloud systems and processes? Addressing these and other related questions is not only of theoretical interest but also of practical interest toward improving tropical variability in low-resolution models used to predict climate and weather.

Cloud-resolving models (CRMs) have been used by a number of investigators as tools for studying tropical convective wave variability (e.g., Oouchi 1999; Grabowski and Moncrieff 2001; Peng et al. 2001; Tomita et al. 2005; Peters and Bretherton 2006; Shutts 2006; Tulich et al. 2007, hereafter TRM07). Invariably, these studies find that large-scale convective wave envelopes (with embedded smaller-scale wave disturbances) tend to develop spontaneously under spatially uniform radiative cooling and zonally uniform sea surface temperatures (SSTs). This is true of expensive three-dimensional (3D) models (e.g., Tomita et al. 2005; Shutts 2006), as well as cheaper 2D models that neglect the effects of planetary rotation, and thus only allow 2D gravity wave motions (e.g., Grabowski and Moncrieff 2001; TRM07).

In a companion study of large-scale waves produced spontaneously in a 2D CRM, TRM07 showed through vertical-mode decomposition that much of the tilted dynamical structures of the waves could be captured by just two dominant vertical spectral “bands,” corresponding roughly to full- and half-wavelength vertical structures in the troposphere. These findings are consistent with a number of observational studies, including Mapes and Houze (1995), Haertel and Johnson (1998), and Haertel and Kiladis (2004). By altering the depth of the model’s convection, as well the background stability profile, TRM07 also gave evidence to suggest that the shallower (and hence, slower) vertical modes play a key role in setting the relatively slow propagation speed of the simulated large-scale waves, in accordance with simple two-mode models of unstable wave growth under the name “stratiform instability” (Mapes 2000; Majda and Shefter 2001). The alternative hypothesis of a reduced-stability, deep-mode mechanism of wave propagation (e.g., Gill 1982; Emanuel et al. 1994), however, could not be ruled out.

In this study, we take a closer look at the 2D CRM simulations of TRM07. Our primary objective is to show how interactions between initially small-scale waves and convection (as seen from a vertical-mode perspective) lead to the spontaneous organization of waves and convection on larger scales. The next section provides a brief overview of the simulations and model analysis methods. Results are given thereafter in sections 3 and 4. Section 5 provides a summary and concluding remarks.

2. Experiment overview

a. Setup

A detailed description of the model and experiment setup can be found in TRM07. Briefly, a 2D version of the System for Atmospheric Modeling (SAM; Khairoutdinov and Randall 2003) is used to perform an ensemble of 5 simulations of spontaneous wave growth from random initial conditions, under a spatially uniform SST of 300 K and a spatially uniform radiative cooling of 1.5 K day$^{-1}$ between the surface and 250 hPa (~10 km), decaying linearly to zero at 200 hPa (~12 km). The domain extends 8192 km in the horizontal and 28 km in the vertical, with periodic lateral boundary conditions and the upper and lower boundaries treated as free-slip, rigid surfaces. To eliminate possible interactions between convection and a domain-averaged flow, damping of the horizontal winds is ap-
plied at all levels with a time scale of 4 h, so that the domain-averaged horizontal winds remain close to zero. Sponge layers are also included in the uppermost 8 km, to provide strong damping of gravity waves reflected off of the upper boundary. For convenience, the 5 simulations are referred to in this paper as runs 1–5, where run 1 corresponds to the “control” simulation given primary attention in TRM07 (see their Figs. 1–12).

b. Patterns of convective organization

All simulations feature spontaneous development of multiscale wave patterns of convective organization. To illustrate the growth and morphology of these patterns, Fig. 2 depicts the space–time evolution of surface precipitation during the first 7 days of run 2. Precipitation onset occurs just prior to the start of day 1, with numerous individual convective systems being distributed at random across the domain. Shortly thereafter (days 1–2), precipitation becomes organized into a series of relatively small-scale \( \mathcal{O}(10–100 \text{ km}) \) wave disturbances or packets (indicated by light shading) that propagate to the left and right at speeds of 12–18 m s\(^{-1}\). The propagation of the packets occurs discretely via the successive generation of one or more contiguous precipitation features with spatial scales in the range 10–100 km and lifetimes of 2–8 h. As time progresses (days 2–5), many of the packets die out, while others persist and in some cases even expand in size. By days 5–7, a series of larger-scale wave envelopes is apparent, with leftward-moving waves dominating over rightward-moving ones for random reasons (cf. TRM07). As with observations in Fig. 1, the large-scale wave envelopes are composed of a crisscrossing array of smaller-scale wave packets, most of which propagate at speeds similar to that of the large-scale envelopes.

c. Discretization of model output

To gain insights into this complex evolution, output from the model is discretized in two ways. First, a vertical-mode transform (detailed in TRM07) projects profiles of horizontal winds \( u \), (virtual) temperature anomalies \( T \), and heating rates \( Q \) onto the vertical modes \( \Psi_n \) of the model’s time-mean sounding. These modes were calculated using the numerical algorithm of Fulton and Schubert (1985), with \( n \) as mode index. Second, a cloud-partitioning algorithm, called the “Zc-Wmax” method, sorts sufficiently cloudy grid columns (i.e., columns with CWP > 0.1 kg m\(^{-2}\), where CWP is the total condensed water/ice path) into three categories: shallow convective, deep convective, and stratus-anvil, where shallow convective columns include precipitating congestus-type clouds, as well as nonprecipitating shallow cumuli (see TRM07 for further details).

Two distinct vertical spectral “bands,” each consisting of a pair of modes, are found to play important roles in the initiation and maintenance of the simulated wave disturbances. The first has vertical structures corresponding to “dry” gravity wave propagation speeds \( c_n \) of roughly 16 and 18 m s\(^{-1}\), while the second has \( c_n \cong 7.7 \) and 8.2 m s\(^{-1}\). Following TRM07, we refer to the first pair of modes as “slow,” to distinguish them from the “fast” modes \( c_n \cong 35 \) and 45 m s\(^{-1}\) most strongly excited by deep convective heating. The second pair are referred to as “gust front” modes, since their propagation speeds are comparable to those of convectively generated surface-based cold pools.

For future reference, Fig. 3 displays the temperature structures of the slow and gust-front modes. The slow modes have their lowest altitude minima at around 2 km, while the gust-front modes have theirs at around 1 km. Both are much lower than the fast modes, which
have their lowest altitude minima in the middle troposphere (see TRM07’s Fig. 8a). Because the horizontal wind profile \( u_h(p) \) of each mode is directly proportional to \(|d T_s(p)/dnp|\), we may associate these lowest altitude temperature minima with the elevation at which the profile of horizontal wind divergence \( \Delta h(p) \) first changes sign.

As already mentioned, TRM07 found that much of the tilted dynamical structures of the large-scale waves could be captured by just two main vertical spectral bands, corresponding to slow and fast modes with roughly full- and half-wavelength structures in the troposphere, respectively. TRM07 also showed that these two bands correspond to distinct peaks in the energy spectra of the large-scale waves, with a much weaker tertiary peak occurring at 11–12 m s\(^{-1}\) (referred to as “ultra slow”; see their Fig. 10). Not emphasized by TRM07, however, was the existence of a fourth peak in the spectra at 7–8 m s\(^{-1}\) (i.e., gust front), whose amplitude was only a small fraction (<5%) of the primary and secondary peaks. Such a small amplitude is consistent with our findings here that the role of the gust-front modes in the initiation of convective wave disturbances is secondary to that of the slow modes. Nevertheless, the fact that the gust-front modes play more of a role in convective wave initiation than the ultraslow (11–12 m s\(^{-1}\)) modes indicates that wave amplitude considerations are not the whole story. Indeed, inspection of TRM07’s Fig. 10a shows that heating processes in deep convective and stratiform regions both provide negative forcing of the gust-front modes (implying low-level destabilization), while the forcing of the ultraslow band is either positive (implying low-level stabilization) or close to zero. Shallow convective heating, meanwhile, provides positive forcing of both spectral bands. In summary, it appears that the sign (as well amplitude) of the wave forcing determines the propensity of a given mode to encourage convective wave organization.

d. Definition/classification of cloud clusters

In addition to the above discretizations, we define “cloud clusters” in the model as contiguous sets of cloudy grid columns (in both \( x \) and \( t \)) that contain two or more deep convective columns. Because our interests lie in how cloud clusters become organized into multiscale wave patterns, a scheme was developed for classifying clusters on the basis of when they develop relative to other clusters. As illustrated in Fig. 4, the

**Fig. 4.** Illustration of a scheme for classifying cloud clusters on the basis of when they develop relative to other clusters. Light, dark, and medium shadings denote columns masked as shallow convective, deep convective, and stratiform, respectively. Sloping lines denote the boundaries of space–time cones (corresponding to phase speeds of \( \pm 19 \) m s\(^{-1}\)) extending backward in time from the earliest set of deep convective columns within each cluster. Solid lines (and contours) emphasize that CC1 is a GEN1 cluster, while dot-dashed and dotted lines emphasize that C2–C5 and C6 are GEN2 and GEN3 clusters, respectively. See text for further details.
scheme starts off by inquiring whether any deep convective columns occur in a space–time cone (bounded by propagation speeds of ±19 m s⁻¹) extending backward in time from each cluster’s earliest set of deep convective columns. Clusters that have no deep convective columns in their backward-looking space–time cones (like CC1 in Fig. 4) are then classified as “first generation” or GEN1 clusters, since they are essentially the first group of clusters to develop. The remaining clusters whose backward-looking cones contain only GEN1 deep convective columns are next classified as “second generation” or GEN2 clusters (e.g., CC2–5), while those that contain only GEN1 or GEN2 deep convective columns are next classified as GEN3 clusters (e.g., CC6), and so on.

3. Interactions between small-scale waves and convection

Using the above diagnostic tools and definitions, we now seek to show how interactions between initially small-scale waves and convection lead to the subsequent development of larger-scale wave disturbances. Our presentation begins with a case study of wave–convection interactions in the context of CC1–4 in Fig. 4. Results from a more general (composite) analysis of wave–convection interactions are then described in section 3b. The mechanisms though which wave–convection interactions leads to upscale wave development are documented and discussed in section 4.

a. Case study

The interaction between waves and convection has two parts, one involving wave generation by convection (primarily through latent heating $Q_{lat}$ plus heating due to turbulent eddy fluxes $Q_{flx}$), and the other involving modulation of convection by waves. To illustrate the former, Fig. 5 shows the time–longitude evolution of fast-mode temperature and horizontal wind signals (using for definiteness the 35 m s⁻¹ mode; denoted by $T_{fast}$ and $u_{fast}$ respectively) over a 350-km-wide region centered on CC1 from Fig. 4. Here, as in TRM07, the sign convention is that $T_n$ and $u_n$ take the same sign of the $n$th modes’s contribution to near-surface temperature and horizontal wind anomalies. Thus, $T_{fast} > 0$ implies a positive contribution to temperature anomalies through the depth of the troposphere (with a peak at around 4 km), while $u_{fast} > 0$ implies a positive contribution to horizontal flow in the lower half of the troposphere, and a negative contribution in the upper half.

The generation of a pair of discrete wave packets by CC1 is clearly evident in Fig. 5. The packets move to the left and right at a speed of 35 m s⁻¹, consistent with theoretical expectations for “dry” gravity modes having $c_n \equiv 35$ m s⁻¹. Inspection of Figs. 6 and 7, together with Fig. 5, shows that the generation of the packets stems from the development of deep convection within CC1, which provides strong heating through a deep tropospheric layer and thus, strong positive forcing of fast-mode temperature anomalies (i.e., $Q_{fast} \gg 0$). Based on theoretical work by Nicholls et al. (1991), such localized forcing (in both $x$ and $t$) is known to excite a pair of horizontally propagating wave packets that transmit positive temperature anomalies generated by the forcing to the environment. As can be inferred from the sign of their horizontal wind signals (Fig. 5b), the packets include flow directed toward (away from) the forcings given in section 2a.
ing in the lower (upper) troposphere and thus, upward (downward) motion toward their leading (trailing) edges. Because the duration of the forcing is \(\sim 1\) h (using \(Q_{\text{fast}} \geq 50\) K day\(^{-1}\) as a threshold), the packets have a zonal extent of \(\sim 125\) km.

In addition to strong heating though a deep tropospheric layer (\(z \equiv 2–12\) km), Fig. 6b shows that CC1’s deep convection provides moderate cooling in the lowest 2–3 km, which stems mainly from evaporation of hydrometeors below elevated cloud bases. This cooling-below-heating provides relatively strong negative forcing of the slow modes, as well as weaker negative forcing of the gust-front modes (Fig. 7b). Shallow convection, meanwhile, acts mainly to warm the lower troposphere, and thus provides moderate-to-weak positive forcing of the slow and gust-front modes (Figs. 6a and 7a).

The impact of these forcings in terms of the temperature responses of the slow and gust-front modes can be seen in Figs. 8a and 8b, respectively. The generation of coherent wave packets by CC1’s convection is once again readily apparent. The packets are smaller in this case, however, owing to their slower propagation speeds. Also, because shallow convective heating (with \(Q_{\text{slow}} > 0\)) precedes deep convective heating (with \(Q_{\text{slow}} < 0\)), as part of the characteristic life cycle of mesoscale convective systems (MCSs; e.g., Mapes et al. 2006), the slow-mode response consists of a succession of two wave packets, the first having positive temperature anomalies and the second having negative anomalies. A similar type of response has been documented in more idealized simulations of growing convective cells by Shige and Satomura (2000) and Lane and Reeder (2001).

Stratiform heating also provides substantial negative forcing of the slow and gust-front modes (at least in area-averaged sense; see Fig. 7c), which lags the deep convective forcing by 1–2 h. In contrast to the deep convective forcing, however, the stratiform forcing is distributed over a relatively broad area, and thus is too weak to excite packets with \(|T_{\eta}| \geq 0.5\) K (contouring threshold in Fig. 8). In other cases, however, we observe large-amplitude waves emanating from the stratiform, as well as deep convective, regions of individual clusters.

Modulation of convection by waves is also apparent in Fig. 8. A prime example is the initiation of CC4’s deep convection (between \(x \equiv 610–620\) km at \(t \equiv 22.5\) h),...
which coincides with the arrival of the slow-mode packet with cold temperature anomalies ($T_{\text{slow}} < 0$) excited by CC1’s deep convection. The passage of the gust-front packets, meanwhile, seems to promote the development of CC2’s deep convection (between $x \equiv 530–540$ km at $t \equiv 23$ h; see also Fig. 9a), as well as deep convection near the left edge of CC1 at $t \equiv 22$ h.

Because the gust-front waves play a key role in modulating convection, it is important to distinguish them from the more widely recognized surface-based cold pools, which also play a role (Nakajima and Matsuno 1988; Tompkins 2001; Liu and Moncrieff 2004). Figure 9 compares the evolution of the gust-front temperature index and near-surface virtual temperature anomalies in the vicinity of CC1 and CC2. Although the gust-front waves have a well-defined gravity wave speed of $\sim 8$ m s$^{-1}$, the cold pools behave more like accelerating “gravity currents,” with speeds ranging from $\sim 3$ m s$^{-1}$ during their early stages to $\sim 7$ m s$^{-1}$ during their later stages. This allows the gust-front waves—whose forcing generally lags that of the cold pools (owing to the upward development of surface-based cooling in association with the MCS life cycle; cf. Fig. 6)—to eventually move out ahead of the cold pools. In accordance with theoretical modeling work (Haertel et al. 2001), the gust-front waves have no discernible temperature signals near the surface.

**b. Composite evolution of GEN1 clusters**

To generalize these results, we document the composite evolution of GEN1 clusters and their associated wave–convection interactions. The reference point of the composite is chosen as the cluster’s strongest

![Fig. 8. Similar to Fig. 5a but for temperature signals of the (a) slow and (b) gust-front modes.](image)

![Fig. 9. Space–time evolution of (a) gust-front-mode temperature index and (b) near-surface virtual temperature anomalies in a 150-km-wide region centered on CC2 from Fig. 4. Heavier contours denote absolute values of 0.25, 0.5, and 0.75 K, with plotting convention similar to Fig. 5, except that lighter contours denote the boundary of CC2, rather than CC1.](image)
deep convective column, as measured in terms of its maximum vertical draft speed $|w_{max}|$. Counts of deep convective columns are also recorded to assess the space–time evolution of deep convection occurrence probability ($P_{dc}$). Since there is no preferred horizontal direction in the model equations (and to increase the statistical robustness of the results), we focus our attention here on the horizontally symmetric component of the composite evolution, given by

$$\langle \varphi(x, t) \rangle_{sym} = \frac{\langle \varphi(x, t) \rangle + \langle \varphi(-x, t) \rangle}{2},$$

where $\langle \varphi \rangle$ is an arbitrary composit field and $x$ is now the horizontal distance from the composite reference point.

Figure 10 depicts the space–time distribution of deep convection occurrence probability $P_{dc}$ for $x \leq 0$. A “fan” of enhanced probability ($P_{dc} \geq 0.75\%$), bounded by propagation speeds of 18 and 4 m s$^{-1}$, extends away with time from the composite reference point. Within the fan, there are two distinct lobes of even larger probability, one centered at a distance of $\sim 50$ km (lobe G) and the other at $\sim 120$ km (lobe S). The orientation of the lobes, together with their relatively large amplitudes ($P_{dc} > 3\%$), indicates that deep convection during the early stages of the simulation is indeed strongly modulated by two distinct wave modes forced by previous cloud systems. This is further supported by Fig. 11 which shows that the axes of G and S reside in the cold sectors of the composite cluster’s gust-front and slow-mode wave responses, respectively. The warm sector of the cluster’s slow-mode response, meanwhile, is a region where convection is suppressed.

### 4. Diagnosis of upscale wave development

Results in the previous section demonstrate that individual cloud clusters during the early stages of runs 1–5 are both strongly modulated by and excite discrete horizontally propagating wave packets with characteristic horizontal scales of 20–80 km and tropospheric vertical wavelengths of $\sim 4$ and 11 km. Here, we extend these results by showing how such interactions lead to the spontaneous organization of waves and convection on larger scales.
a. Composite analysis of later cluster generations

A composite analysis of cluster generations 2 and 3 serves to illustrate the mechanism of upscale wave development in its simplest form. To aid the analysis, we introduce the variables:

\[ R_n = T_n - \alpha_R u_n \quad \text{and} \quad L_n = T_n + \alpha_R u_n, \]

which convey the amplitude (and sign) of rightward- and leftward-moving wave signals, respectively. Here, \( \alpha_R \) is a units conversion factor that gives \( R_n \) and \( L_n \) the units of temperature (see appendix for details). Clusters can then be further categorized based on whether they develop in a wave field dominated by rightward- or leftward-moving signals, assessed here using the quantities, \( R_{\text{slow}} \) and \( L_{\text{slow}} \). The former (latter) represents the root-mean-square amplitude of rightward- (leftward-) moving slow-mode signals in the cluster’s backward-looking space–time cone, extending out to a time lag of –6 h. To double the statistical sample, we flip all scalar and vector fields associated with clusters dominated by leftward-moving waves (i.e., \( L_{\text{slow}} > R_{\text{slow}} \)) and composite them in the rightward-moving depiction of Fig. 12.

A discrete, 2-stage wave-expansion process, involving the successive formation of at least two distinct cloud clusters (later referred to as “parent” and “child”), is suggested by the composite evolution in Fig. 12. During the first stage, a relatively narrow slow-mode wave packet with \( T_{\text{slow}} < 0 \) (shading) excited by the parent cluster eventually reaches the child cluster, causing its local environment to become substantially cooler in the lower free troposphere (see Fig. 13a). The cooling increases cloud buoyancy by definition and apparently together with moisture effects (see Fig. 13b; see also Kuang 2008) promotes the growth of shallow convection into deep convection over a period of ~30 min.

During the second stage, strong positive forcing of the slow modes by the child cluster’s shallow convection (negative lags in Fig. 14) leads to an erosion (or complete damping) of a small section of the ambient wave some distance behind its leading edge (e.g., the clear gap between shaded fingers in Fig. 12a). This is followed by strong negative forcing of the slow modes by deep convective and stratiform heating, with the former leading the latter by roughly an hour or more. The persistent negative forcing leads to an intensifica-
tion and widening of the ambient wave as it passes through the child cluster. Meanwhile, weaker negative forcing of the ambient wave (indicated by dashed line in Fig. 15a), however, convective signals associated with R1 imply a propagation speed close to the dry wave speed. Inspection of Fig. 16 shows that this difference can be attributed to differences in the morphologies of clusters associated with L1 and R1. Specifically, two of the clusters in L1 provide anomalously large damping of the cold phase of the wave by shallow convective heating (indicated by arrows), resulting in delayed development of subsequent clusters (i.e., later in time).

Another mechanism for upscale wave development is through the horizontal expansion of one wave disturbance into another. For example, Fig. 17 shows how the development of the disturbance L4 stems from the “merger” of two smaller-scale disturbances L2 and L3. Later on, we see that L4 merges with L1 to form an even larger-scale disturbance. We emphasize that this type of wave expansion, as well as the one described earlier, relies on the tendency of convection (forcing) to persist locally after the ambient wave has moved through the convective region.

Examples of decaying waves can also be found in Figs. 15 and 17. Presumably, some of this decay stems from the effects of parameterized/numerical diffusion, which tend to damp wave dynamical signals, with the smallest-scale signals being damped most rapidly. Additionally, there are numerous shallow convective clouds in the model whose buoyancy is too weak to allow their upward development into deep convective clouds with stratiform anvils (not shown). The heating associated with these shallow clouds causes further damping of cold wave envelopes, so that wave disturbances that provoke only shallow convection sow their own demise.

c. Analysis of fast-moving wave signals

Shifting our attention to the fast modes for completeness, Fig. 18 depicts the space–time evolution of leftward- and rightward-moving wave signals in the vicinity of Fig. 15’s L1 and R1. The radiation of discrete wave packets by individual clusters is similar to that discussed earlier in the context of CCI—the sign of the packets imply deep-tropospheric warm temperature anomalies, with wind directed toward (away from) the convective heating in the lower (upper) troposphere; that is, deep convection causes large-scale, deep-tropospheric ascent.

As a more general assessment of these fast-mode-
convection interactions, Fig. 19 shows results from a Fourier cross-spectrum analysis of fast-mode heating $Q_{\text{fast}}$ and horizontal wind convergence $-\delta_{\text{fast}}$ in the wavenumber-frequency domain (e.g., Wheeler and Kiladis 1999). The enhanced power along phase lines of 14–16 m s$^{-1}$ has sometimes been interpreted in terms of deep-mode wave ascent causing moist heating, which reduces the effective static stability and thus the phase speed of the wave (e.g., Gill 1982; Emanuel et al. 1994). However, close inspection of the imaginary part of the spectrum (arrows) shows that $Q_{\text{fast}}$ typically leads, rather than lags, $-\delta_{\text{fast}}$ by a small amount. The implication is that deep-tropospheric ascent in the model is primarily a consequence, rather than a causal agent, of deep convection.

d. Simulations with prescribed modal wind damping

Although the fast modes do not appear to play a role in the generation and propagation of mesoscale wave disturbances, their potential role in fostering the organization of convection into larger-scale wave envelopes
cannot be ruled out. The evolution of fast-mode wave signals in Fig. 18, for example, illustrates how deep convective heating associated with one wave disturbance can potentially suppress convection associated with surrounding wave disturbances, via deep-tropospheric stabilization (as in the empty front edge of L1 in Fig. 15). This “communication” among wave disturbances could lead to a kind of natural selection process, whereby incipient disturbances with larger (smaller) amplitudes are more (less) likely to persist and expand horizontally with time.

To explore this idea, we performed an additional simulation in which the deep-tropospheric components of horizontal wind anomalies (specifically, those with $c_n > 30$ m s$^{-1}$) were artificially damped, so that amplitudes of fast-moving wave packets decrease with distance from their source regions, inhibiting communication among wave disturbances. The damping was applied via Rayleigh relaxation with a 2-h time scale, implying an $e$-folding (decay) distance of several hundred kilometers for the most strongly excited wave modes ($c_n \equiv 35$–60 m s$^{-1}$). Results in this case show

Fig. 16. Depiction of finescale structures of disturbances L1 and R1 from Fig. 15. Shading denotes regions where (a) $L_{\text{slow}}$ and (b) $R_{\text{slow}}$ fall below a threshold of $-0.3$ K, with contour increments of 0.1 K. Thin contours denote the boundaries of individual cloud clusters, with deep convective regions filled black. As in Fig. 15, thick solid and dotted lines outline the cold wave envelopes of L1 and R1, respectively. Arrows in (a) denote two occasions where strong positive forcing of the slow modes by individual clusters within L1 leads to strong damping of incoming wave signals, resulting in delayed development of subsequent clusters and hence, a reduction in convective envelope propagation speed (as compared to a dry wave speed of 16–18 m s$^{-1}$).

Fig. 17. Similar to Fig. 15a but for a larger subdomain, where thick dashed lines denote the cold wave envelope of (top) a leftward-propagating disturbance L4 that develops from the merger of two smaller-scale envelopes: (bottom) L2 and L3.

Fig. 18. Similar to Fig. 15 but for fast-mode (35 m s$^{-1}$) wave signals (a) $L_{\text{fast}}$ and (b) $R_{\text{fast}}$. Only absolute values greater than 0.2 K are shaded with contour intervals of 0.2 K.
These findings are reminiscent of the study by Liu and Moncrieff (2004), who showed how large-scale wave organization in 2D CRMs can be suppressed by including the effects of planetary rotation, so that the gravity wave response to convective heating is essentially trapped at a finite distance from the heating.

Figure 20b illustrates the importance of shallower vertical wavelengths in setting convective wave propagation speeds. Shading denotes precipitation in a run with modal $u$-wind damping applied over the range $c_n = 14-25 \text{ m s}^{-1}$. Although multiscale convective wave patterns are still apparent, their propagation speeds are now much smaller, being comparable to those of the gust-front modes. Apparently, damping of the more strongly excited slow modes leads to convection coupling to the more weakly excited gust-front modes. This is supported by Fig. 21, which shows that gust-front wave packets now play a leading role in convective triggering.

5. Summary and conclusions

This paper described an objective analysis of multiscale convective wave patterns produced spontaneously in a 2D CRM under spatially uniform radiative cooling. As glimpsed in previous modeling efforts (e.g., Oouchi
stabilization by deep-tropospheric circulations ($L_{cn}$ = 2D), as well as shallower circulations, is important for the clustering of mesoscale convective wave disturbances into larger-scale wave envelopes.

Nonlinear effects such as transports of horizontal momentum by coherent eddy circulations may also play an important role in the dynamics of tropical convective waves, as postulated, for example, by Moncrieff (2004) and Biello and Majda (2005). From the perspective of the vertically transformed system (A1–2), such transports can be treated as an apparent source term in the horizontal momentum equation, which could provide forcing of atmospheric wave circulations, even in the absence of latent heating. Although this type of wave driving was generally found to be small in our idealized simulations (results not shown), more systematic effects could arise in nature under conditions of strong vertical shear of the background flow.

The relevance of this study’s findings toward lower-frequency tropical disturbances such as the Madden–Julian oscillation (MJO) is not yet clear. As is fairly well known, the dispersion properties of the MJO are unlike those of any shallow-water equatorial waves, suggesting that the oscillation is more than just a simple linear wave instability (e.g., Hendon 1988; Raymond 2001). Nevertheless, recent observations of the temperature and moisture structures of the MJO (Kiladis et al. 2005; Benedict and Randall 2007) show interesting fluctuations in the lower free troposphere at around 800–900 hPa, similar to the gust-front wave circulations described here. Could it be that these shallow fluctuations (together with more exotic nonlinear processes, such as horizontal moisture transports by Rossby-gyre circulations) are somehow related to the relatively slow (~5 m s$^{-1}$) eastward propagation of the MJO? Similarly, how do interactions between clouds, water vapor, and radiation impact the forcing and maintenance of shallow and deep circulations associated with the MJO? Processing of data from several recent field experiments (e.g., May et al. 2007), as well as output from more realistic 3D models (e.g., Grabowski 2003; Khouider et al. 2005; Nasuno et al. 2007), may help to address these questions more fully.

Acknowledgments. We thank George Kiladis for his many helpful discussions and for pointing out the remarkable high-frequency convective wave disturbances seen in the Cloud Archive User Service (CLAUS) data. Comments by Mitch Moncrieff and one anonymous reviewer lead to improvements in this manuscript. This research was supported by the National Science Foundation under Grant ATM-0555570.
APPENDIX

Directional Decomposition of Wave Signals

In the absence of rotation, equations describing the horizontal evolution of the nth vertical mode may be written as

\[
\frac{\partial T_n}{\partial t} - c_n \alpha_R \frac{\partial u_n}{\partial x} = 0 \quad \text{and} \quad \frac{\partial u_n}{\partial t} - c_n \alpha_R \frac{\partial T_n}{\partial x} = Q_n, \tag{A1}
\]

where \( T_n, Q_n, \) and \( u_n \) are amplitude indices for temperature, heating, and horizontal wind perturbations, respectively, \( \alpha_R = 1 \, \text{K} \, \text{m}^{-1} \, \text{s} \) is a units conversion factor (given \( T_n \) the units of temperature), and the effects of horizontal momentum forcing have been neglected for simplicity (see TRM07 for further details). Introducing the variables

\[
L_n = T_n + \alpha_R u_n \quad \text{and} \quad R_n = T_n - \alpha_R u_n, \tag{A2}
\]

(A1) and (A2) can be manipulated to obtain

\[
\frac{\partial L_n}{\partial t} - c_n \frac{\partial L_n}{\partial x} = Q_n \quad \text{and} \quad \frac{\partial R_n}{\partial t} + c_n \frac{\partial R_n}{\partial x} = Q_n. \tag{A5}
\]

In the absence of forcing, these equations are nothing more than a pair of linearly independent advection equations describing wave signals moving to the left and right, respectively, at constant gravity wave speed \( c_n \).

REFERENCES


