Dispersion analysis of Discontinuous Galerkin schemes applied to Poincaré, Kelvin and Rossby waves

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Abstract

We propose here a technique for analysing dispersion properties of high order Discontinuous Galerkin Schemes in two dimensions for the system of rotating shallow water equations. The method can be applied to non regular grids and is able to deal with non periodic boundary conditions. The method can also be applied to differential operators for which continuous eigenfunctions are not sines and cosines. An eigenvalue analysis of the discrete operator is performed. An error estimator is initially applied to each eigenfunction in order to distinguish resolved and unresolved modes. Some kind of Fourier analysis is applied to the eigenfunctions in order to determine the numerical wave number spectrum. For resolved modes, the discrete dispersion relation is constructed using the wavenumbers (eigenfunctions) and frequencies (eigenvalues). Both spatial and spectral convergence properties are found out in one analysis. For example, the 1D Radau structure of the DG spatial error is clearly exhibited. The method is applied to the discretization of rotating shallow water equations. The β-plane approximation is used to take into account a Coriolis force with variable angulus momentum. Such a model exhibits three families of dispersive waves, including the slow Rossby waves that are usually difficult to analyse. Rossby waves are usually treated using a WKB *(Wentzel-Kramers-Brillouin)* approximation[1] of the coriolis term (slowly varying coefficients). In our case, the WKB approximation is not accurate enough for a convergence analysis: the DG scheme is so accurate that its dispersion error is below the WKB error. Here exact Rossby frequencies are computed solving the Longuet-Higgins equation (that only involves the y component of transport) and using a high order 1D scheme. We show that the DG scheme is super-accurate at order 2p+3 for the Rossby waves. Channel and equatorial Kelvin waves only exist in presence of a wall boundary condition and are therefore out of reach of classical dispersion analysis. Our method allow to demonstrate that the DG method is able to predict Kelvin waves with a super-accuracy (order 2p+3).

[1] A. Majda, *Introduction to PDE's and waves for the atmosphere and ocean*, AMS, 2003.