

# How to ensure mass conservation in a three-dimensional, free-surface, finite-element marine model on a moving mesh

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### Abstract

Large-scale free-surface ocean models designed to run over climatic timescales are required to globally conserve the volume and any tracer up to machine precision. In addition, the following property is critical. Setting a tracer to a uniform value throughout the closed domain and letting the free surface undulate, we must recover the same tracer concentration at any later time. This property of *local conservation* ensures that no spurious extremum is created. We present a three-dimensional, free-surface finite-element marine model that conserves any tracer locally and globally. The mesh is unstructured in the horizontal and is allowed to move in the vertical to track the free-surface motions. We show that local conservation requires a discrete compatibility between the tracer and continuity equations and that local conservation does not necessarily lead to global conservation. To have both local and global conservation, we show that a discrete compatibility between the tracer, continuity and free-surface equations must be fulfilled. It is suggested that this compatibility constraint, together with the use of a stable scheme, severely restricts the choice of spatial discretizations. Some test cases are presented where the method is shown to satisfy all conservation properties. Future improvements are outlined.