

# The relation between RT0 finite element and circumcentred finite volume methods for the shallow water equations

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In the unstructured grid modelling of coastal, shelf and ocean flows both finite volume and finite element methods have been successfully applied. In Miglio et al. (1999) a connection is made between the RT0 FEM discretisation of the shallow water equations (such as in e.g. Walters and Casulli (1998)) and the FDM/FVM discretisation as in Casulli and Cheng (1992). It is shown that the latter can be derived from a unique first order diagonalisation of the mass matrix in the RT0 discretisation. In this presentation we will show that this not only holds for structured rectangular grids but also for the circumcentred unstructured grids as in Casulli and Walters (2000).

This connection may provide new insights in the properties of the schemes and prove useful in the discretisation of Coriolis and advection terms. In Ham et al. (2006) it is shown that for the discretisation of the Coriolis term in unstructured circumcentred schemes a special weighting needs to be applied in order to obtain a stable and energy conservative scheme. Here we will show how this property can also be derived directly from the first order approximation of the RT0 scheme. The same procedure can also be applied to the discretisation of the advection terms. The result of this can be related to the conservative staggered schemes found in Perot (2000).

## References

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