1. **numerical gravity wave dispersion**

   The tide propagation inside a channel is simulated using a finite difference model. The user is trying to decide how to configure it. Use the dispersion relationship to decide on the number of points needed per wavelength to represent the phase speed at better than 95% accuracy. Use the dispersion relations to estimate this number for staggered and unstaggered grids of 2nd, 4th and 6th order. Repeat the exercise for 99% accuracy. You can use Matlab’s `solve` command to solve nonlinear equations. Alternatively, you can use the Fortran code `newtonraphson.f90` which is downloadable from the website.

2. **numerical Rossby wave dispersion**

   A climate simulation is run on a 1° grid using a second order differencing scheme on a C-grid. Assuming a Rossby radius of deformation of about 50 km at 45 degree latitude, a $\beta = 2 \times 10^{-11} \text{m}^{-1} \text{s}^{-1}$ and $f = 10^{-5} \text{s}^{-1}$, how well are zonal ($l = 0$) Rossby wave speeds represented if their wavelengths is 50, 100, 200 km? Remember that the zonal grid spacing varies like $R \cos \theta$ with latitude, where $R$ is the Earth’s radius which can be taken as $R = 6370 \text{km}$. What happens if the resolution is increased to 1/12°?

3. **1D SWE code**

   The linearized shallow water equations are given by

   \[
   u_t + g \delta_x \eta = 0 \quad (1) \\
   \eta_t + \delta_x (U) = 0 \quad (2)
   \]

   where $U = \mathbf{H}^T u$ is the (linearized) mass transport in the x-direction.

   (a) Show that, in the continuous form, the energy, $E = Hu^2/2 + g\eta^2/2$, is conserved when the boundaries are closed: $(\int_L E dx)_t = 0$.

   (b) Show that in the spatially-discrete form, the energy defined as

   \[
   E_j = H \left( \frac{(u^2)^j}{2} + \frac{\eta_j^2}{2} \right)
   \]

   is also conserved.

   (c) Write a code to solve the linearized 1D SWE equations: on a C-grid using second order centered difference approximations. Assume a channel with closed ends, and a constant still water depth $H$.

   (d) Apply the code to simulate the sloshing of a standing wave system with the following data:

   Domain: $0 \leq x \leq 1$

   IC: $u(x,0) = 0$ and $\eta(x,0) = \cos 4\pi x$

   BC: $u(0,t) = 0$ and $u(1,t) = 0$
data: $g = 1$ and $H = 1$.

exact solution: $u(x,t) = \sin 4\pi x \sin 4\pi t$. $\eta = \cos 4\pi x \cos 4\pi t$.

Your report should contain the following information

- an estimate of the stability restriction associated with the time-stepping scheme; the latter’s choice is up to you. Please use the stability diagrams from the note to come up with the limits to the maximum time-steps.
- A confirmation that the scheme is second-order as you refine the grid spacing
- A confirmation that the mass, $\sum_{j=1}^{N} \eta_j$, is conserved to machine precision.
- A confirmation that the energy, $\sum_{j=1}^{N} E_j$, where $E_j$ is defined in equation, is conserved up to errors in the time-stepping scheme.

In writing your code you should re-use as much as possible from the codes you have already written, whether it is the ODE solver or the grid partitioner, or the advection code.