

Research and Teaching Interests for Francisco Javier Beron-Vera *

1 Research Interests

My research efforts to date have combined both theoretical and observational aspects of geophysical fluid dynamics. Specific topics that I have investigated include: large-scale ocean dynamics and thermodynamics; geophysical waves and flow stability (spectral, formal, and nonlinear); Hamiltonian geophysical fluid dynamics; nonintegrable Hamiltonian dynamics; passive tracer transport and mixing with applications to the Earth's oceans and atmosphere, and planetary atmospheres; underwater acoustic-ray dynamics. The research papers that resulted from investigation on these topics are briefly described below. Further details as well as reprints can be found in my website at <http://www.rsmas.miami.edu/personal/fberon>.

M.Sc. and Sc.D. Research

Both my M.Sc. and Sc.D. studies were conducted at CICESE (Ensenada, Baja California, Mexico) under the supervision of the late Prof. Pedro Ripa. These studies resulted in the publication of four research articles (Beron-Vera and Ripa 1997, 2000, 2002; Beron-Vera et al. 1999) which are summarized below.

The paper Beron-Vera and Ripa (1997), which constituted my M.Sc. thesis, deals with the effects of a freely moving boundary on quasigeostrophic (QG) baroclinic instability assuming a reduced-gravity setting. Substantial differences with respect to the standard (i.e., horizontally rigid boundary) QG baroclinic instability problem were found by thoroughly inspecting the space of basic state parameters. For instance: (i) free boundary QG baroclinic instability possesses both high and low wavenumber cutoffs; (ii) the growth rate of the most unstable wave can be twice as large as that of the most unstable Eady wave; (iii) the wavelength of this wave has a scale that can be much larger than the Eady scale; and (iv) instability may be inhibited when the interface and isopycnals slope in the same direction, whereas it is possible to have instability for arbitrary short waves when they slope in opposite directions.

In Beron-Vera and Ripa (2000), which was written as part of my Sc.D. thesis, we studied the seasonal heat balance in the Gulf of California using a minimal box model suggested by historical hydrographic data. The model allowed us to study of three-dimensional aspects of the balance. Strong vertical heat fluxes were found in the relatively small region of the gulf occupied by the Ballenas Channel, reinforcing previous speculations that in such region vertical mixing should be very intense. The Pacific Ocean fluxes estimates at the annual frequency were found to agree well with those associated with the propagation of a baroclinic wave excited at the mouth, which was previously shown to explain a big deal of the seasonal heat balance in the gulf.

Also as part of my Sc.D. thesis, in Beron-Vera and Ripa (2002) we investigated the seasonal balance of the average salinity in the Gulf of California, which, unlike heat, presented a very important semiannual signature. A box model was employed to estimate the longitudinal (i.e., along-gulf) residual salinity flux and a linear one-dimensional two-layer model, with an inhomogeneous slab-like upper layer, to explain the underlying physics at the annual and semiannual frequencies. The Pacific Ocean was found to control a large extent of the average salinity balance at these frequencies by exciting a baroclinic wave at the mouth of the gulf, thereby confirming the importance of the Pacific Ocean in the dynamics and thermodynamics of the gulf.

Finally, in Beron-Vera et al. (1999) surface boundary conditions for use in salt and freshwater balances were considered. This investigation was motivated by a desire to clarify the concepts involved in the derivation of the boundary conditions. This required to distinguish the flux of a property from its transport across

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a (possibly moving) surface. The implications of this distinction were discussed in the paper. Also, a systematic derivation of consistent approximations was provided.

Postdoctoral Research

I acquired postdoctoral training under the supervision of Prof. Michael G. Brown at RSMAS (University of Miami). The general subjects of research in this position were the study of ray dynamics in long-range underwater sound propagation, and the stability of Lagrangian particle trajectories in incompressible two-dimensional flows. Both ray and fluid particles trajectories obey a 1.5-degree-of-freedom Hamiltonian system, which allowed us to exploit ideas relating to Hamiltonian chaos.

Ray stability was investigated in environments consisting of a range-independent background sound-speed profile with a very highly structured range-dependent perturbation, such as that produced by internal waves in deep ocean environments. We provided numerical evidence of (i) the occurrence of a mixed phase space wherein regular and chaotic ray trajectories coexist, and (ii) the influence of the background structure on ray stability. Shear-induced instability enhancement was proposed as the basic mechanism through which the background sound-speed structure drives ray instability. We also investigated the effects of the background field on the stability of timefronts. The same mechanism was found to control timefront stability. This investigation resulted in four published papers (Beron-Vera et al. 2003; Beron-Vera and Brown 2003, 2004; Brown et al. 2005) and one further paper that is nearing completion (Beron-Vera and Brown 2008).

In this position I have also studied the stability of Lagrangian particle trajectories in steady basin-scale gyre-like wind-driven ocean circulations on which wave-like perturbations are superimposed. The partitioning of phase space (i.e., physical space) into regular motion regions and chaotic motion regions was found to apply to a large class of unsteady flows, including flows that are not close to steady. The latter leading to patchiness in passive tracer distributions, i.e., distributions that are mostly vigorously stirred but include generally poorly stirred regions. For flows that are close to steady, trajectory stability was largely controlled by the background (steady) flow. A shear-induced instability enhancement argument, which arises naturally in the fluid mechanics context, was used to explain the influence of the background flow on trajectory stability. This work resulted in one publication (Beron-Vera, Olascoaga and Brown 2004).

Some time was reserved in this position for pursuing other interests. This resulted in the preparation of five research articles, four of which are published (Beron-Vera and Olascoaga 2003; Olascoaga et al. 2003; Beron-Vera 2003; Beron-Vera, Olascoaga and Zavala-Garay 2004) and one is currently under revision (Beron-Vera 2007). The main results from these articles are described below.

In Beron-Vera and Olascoaga (2003) we investigated the potential of baroclinic instability in the Atlantic North Equatorial Current (ANEC) using a quite general (inviscid, unforced) 3-layer QG model. Employing data from the region occupied by the current, we evaluated the conservation laws-related stability properties of a baroclinic zonal flow in that model. The ANEC model resulted unstable to normal-mode perturbations, reflecting that it is far too wide for Arnold's second theorem to hold, i.e., no negative-definite pseudo energy-momentum integral was possible to be found. The instability of the current was interpreted in terms of the resonant interplay of certain vorticity-related and Rossby-like modes. An important conclusion of this investigation was that the much simpler 2-layer reduced-gravity model fails to predict instability for the flow.

In Olascoaga et al. (2003) we extended earlier results regarding the effects of the lower layer of the ocean (below the thermocline) on the baroclinic instability within the upper layer (above the thermocline). We confronted QG baroclinic instability properties of a 2.5-layer model with those of a 3-layer model with a very thick deep layer, which was previously shown to predict spectral instability for basic state parameters for which the 2.5-layer model predicts nonlinear stability. We computed and compared maximum normal-mode perturbation growth rates, as well as rigorous upper bounds on the nonlinear growth of perturbations

to unstable basic states, paying particular attention to the region of basic state parameters where the stability properties of the 2.5- and 3-layer model differ substantially. We found that normal-mode perturbation growth rates in the 3-layer model tend to maximize in this region. We also found that the size of state space available for eddy-amplitude growth tends to minimize in this same region. Moreover, we found that for a large spread of parameter values in this region the latter size reduces to only a small fraction of the total enstrophy of the system, thereby allowing us to make assessments of the significance of the instabilities.

In Beron-Vera (2003) I gave a derivation of Salmon's balance model for rotating shallow-water flow—an important prototype of constrained Hamiltonian balance model—in full spherical geometry. This was done by performing approximations in Hamilton's principle for shallow-water dynamics, wherein variations are taken on Lagrangian particle labels at fixed Eulerian positions and time. The investigation, which was initiated while I was a Sc.D. student, was motivated by the fact that almost all derivations of Hamiltonian balance models reported in the literature are done using the Cartesian coordinates of the β plane approximation, which is known to be inconsistent in geographical coordinates.

In Beron-Vera (2007) I generalized Ripa's inhomogeneous-density single-layer primitive-equation model to include an arbitrary number of layers. Ripa's model represents a very important improvement over bulk mixed layer models. Namely, those models that consider velocity and buoyancy fields as vertically uniform or slablike, or that have an implicit representation of the vertical velocity shear through the thermal wind balance with the lateral buoyancy gradient. In addition to allowing arbitrary velocity and buoyancy variations in horizontal position and time, in Ripa's model these fields are also allowed explicitly to vary linearly with depth. As a consequence, Ripa's model enjoys a number of properties which make it to fare much better than bulk mixed layer models.

Finally, in Beron-Vera, Olascoaga and Zavala-Garay (2004) we showed that substantially more accurate overall results in free-boundary QG baroclinic instability than those produced by the single-layer version of Ripa's model can be attained by considering my extension of this model with the inclusion of only a few layers. The preliminary results of free-boundary ageostrophic baroclinic instability presented in (Beron-Vera 2007) also looked very promising.

Current Research

I currently serve as PI of the NSF funded project CMG-0417425 which focuses on two sets of problems in ocean physics in which the underlying dynamics are those of a nonintegrable Hamiltonian system. The problems being investigated in this project, which has supported one PhD student, are: (i) particle trajectory (Lagrangian) dynamics in unsteady two-dimensional incompressible flows, and (ii) wave propagation in inhomogeneous moving media. Traditional approaches to the above two classes of problems rely heavily on strictly stochastic methods. From a dynamical systems perspective such an approach is unrealistic and unnecessarily restrictive.

Eight research papers are either published, submitted, or nearing completion on this project that relate to problem (i) (Olascoaga et al. 2006; Olascoaga, Beron-Vera, Brand and Koçak 2008; Olascoaga, Beron-Vera, Brown and Koçak 2008; Rypina et al. 2007a,b; Beron-Vera, Brown, Olascoaga, Rypina, Koçak and Udovydchenkov 2008; Beron-Vera, Olascoaga and Goni 2008; Beron-Vera and Olascoaga 2008).

In Rypina et al. (2007a) we deal with Lagrangian dynamics of atmospheric zonal jets and the permeability of the stratospheric polar night vortex. In this paper we argue that a twistless KAM torus—invariant material closed curve for which the standard nondegeneracy condition used in Kolmogorov–Arnold–Moser theory is violated—accounts for the sharp boundary of the Antarctic ozone hole at the perimeter of the stratospheric polar vortex in the austral spring, thereby providing an explanation for which ozone-depleted air does not spread via turbulent mixing to midlatitudes.

In Rypina et al. (2007b) we address the remarkable stability of twistless tori under perturbation, which is shown to be linked to very small resonance widths near these tori. We have introduced the term *strong*

KAM stability to designate the mechanism by which twistless tori result so resistant to break up under perturbation. In addition to passive tracer transport in the ocean and atmosphere, twistless tori arise in many other applications including simple mechanical systems, charged particle dynamics in magnetic fields, celestial dynamics, stellar pulsations, plasma physics, and underwater acoustics.

The recently accepted paper Beron-Vera, Brown, Olascoaga, Rypina, Koçak and Udovydchenkov (2008) paper deserves a longer discussion. Transport barriers in geophysical flows have been traditionally associated with sharp gradients in the distribution of the ambient potential vorticity (PV). Theoretical and numerical work has demonstrated that QG β -plane (or hemispherical) turbulence tends to evolve to a strongly anisotropic state dominated by alternating narrow eastward and broad westward zonal jets with almost piecewise constant PV between adjacent eastward jets. The atmosphere of Jupiter provides a vivid example of a mean flow pattern with these qualitative features. Observations of Jupiter strongly suggest that both eastward and westward jets act as robust meridional transport barriers, which cannot be explained using the traditional PV-gradient criterion. Contrarily, our strong KAM stability mechanism successfully predicts that both eastward and westward zonal jets should act as robust meridional transport barriers. The numerical simulations of an idealized model of Jupiter's midlatitude circulation at the cloud top level presented in Beron-Vera, Brown, Olascoaga, Rypina, Koçak and Udovydchenkov (2008) show that both eastward and westward zonal jets act as robust meridional transport barriers, consistent with the strong KAM stability explanation of the transport barrier mechanism. Further details can be found in my website at <http://www.rsmas.miami.edu/personal/fberon/jupiter/Jupiter.pdf>.

Work in preparation includes completion of a review article (Beron-Vera 2008) on the dynamics associated with nonautonomous quasiperiodic perturbations to integrable autonomous Hamiltonians. This includes a proof of the admission of invariant tori in the case that the unperturbed Hamiltonian does not satisfy the standard Kolmogorov nondegeneracy (or twist) condition, but rather the weaker condition due to Rüssmann. The issues of nonautonomous quasiperiodic perturbations and degeneracy are central in all of the aforementioned works.

In Olascoaga et al. (2006) we deal with Lagrangian dynamics on the West Florida Shelf (WFS). Using modeled surface currents, in this work we identify *Lagrangian coherent structures* (LCSs) on the WFS. LCSs produce maximizing curves in *finite-time Lyapunov exponent* (FTLE) fields, and delineate past and future histories of boundaries of fluid domains with distinct advective properties. The simulated LCSs reveal the presence of a cross-shelf transport barrier in approximately the same location as the western boundary of the so-called "forbidden zone," which is a region on the southernmost part of the WFS that was found earlier not to be visited by drifters that were released outside of the region. In addition to being an interesting physical feature whose underlying dynamics deserves further study, the cross-shelf transport barrier on the WFS is argued to have potentially very important biological implications.

Indeed, in Olascoaga, Beron-Vera, Brand and Koçak (2008) we exploit the fluid particle motion information contained in LCSs to trace the origin of an observed red tide on the WFS. More precisely, in that paper we employ LCSs to trace the early location of a red tide on the WFS before it was transported to an area where it could be detected by satellite imagery, and then make use of a population dynamics model to infer the factors that may have led to its development. We find that the evolution of the satellite-tracked red tide is strongly tied to the simulated LCSs.

In close connection with the above biophysical modeling work, in Olascoaga, Beron-Vera, Brown and Koçak (2008) we investigate basic aspects of plankton dynamics in the ocean. More specifically, in that paper we study the effects of Lagrangian mixed phase space dynamics and population dynamics in plankton patchiness generation.

In Beron-Vera and Olascoaga (2008) we carry out a detailed study of the nature of tracer evolution in the WFS. Using several diagnostics we find that the concept of "chaotic stirring," understood here as a situation wherein the advection field is spatially coherent and temporally quasiregular on timescales over which the fluid particle motion is irregular, is relevant for the evolution of tracers passively advected by the same

surface ocean velocity field that sustained the aforementioned persistent cross-shelf transport barrier on the WFS.

In Beron-Vera, Olascoaga and Goni (2008) we demonstrate the feasibility of detecting meaningful LCSs in surface ocean currents inferred using climatological hydrography and altimetry by computing FTLEs. Detection of LCSs, which cannot be revealed by visual inspection of velocity field snapshots, provides an objective (i.e., frame-independent) means of identifying mesoscale vortex boundaries and quantifying transport by these vortices which does not rely on the generally incorrect assumption that transport is effected by the trapping and subsequent translation of fluid slugs inside closed sea surface height contours within which rotation dominates strain. For the reader's convenience an animation comparing the trajectory of a satellite-tracked drifter with the pathways determined by attracting LCSs (i.e., maximizing ridges in backward-time FTLE fields) has been made available at http://www.rsmas.miami.edu/personal/fberon/lcs/wal_dftle_bwd60d.avi.

Two research articles that relate to problem (ii) above are published or nearing completion (Beron-Vera and Brown 2008; Brown et al. 2005) which have already been discussed. Future planned work on this problem includes the development of a theory of wave propagation in random inhomogeneous media; preliminary work on this problem suggests that it differs in some fundamental respects from the traditional (homogeneous background) problem.

Future Research Plans

I submit in an almost continuous basis research proposals to various funding agencies including NSF and NASA. These proposals currently concern applications of nonintegrable Hamiltonian systems and LCS theory.

One proposal submitted to NSF (CMG0825547) has been recently awarded which represents an extension of the above-described ongoing NSF-sponsored (CMG0417425) collaboration between geoscientists and a mathematician whose focus is applications of nonintegrable Hamiltonian systems to problems in geophysical fluid dynamics. The principal mathematical tools applied in this effort are results relating to Kolmogorov–Arnold–Moser theory (which addresses the stability of Hamiltonian systems under perturbation) and results relating to the structure of stable and unstable manifolds of hyperbolic trajectories in nonsteady flows. These manifolds are often referred to as Lagrangian coherent structures, or LCSs, in fluid dynamical applications. KAM theory and manifold structure are intimately linked. Applications on which we will focus are: (1) the connections between jets, transport barriers and potential vorticity barriers in the Earth's oceans and stratosphere; (2) LCS climatology associated with the general circulation of the ocean and the connection between these structures and the predominant Eulerian features of the general circulation; (3) biological applications of oceanic LCSs including problems involving harmful algal blooms, plankton patchiness, and understanding observed biogeographical boundaries; and (4) problems involving a dynamical systems approach to wave propagation in random inhomogeneous media.

The main goal of a proposal which is planned to be resubmitted to the NASA Ocean Topography Team is to employ the FTLE method to identify and subsequently track mesoscale eddies in the ocean. The FTLEs will be computed using surface geostrophic velocities inferred from climatological hydrographic observations and altimetric sea height measurements. The FTLE method resolves the subjectivity associated with the traditional approach to eddy boundary identification, which has relied on visual inspection of sea surface topography maps, and the more recent Eulerian eddy identification criteria, such as the Okubo–Weiss criterion. The FTLE method allows, for instance, to differentiate the eddies shed from the main current, which is critical to assess the current and eddies exact contribution in the exchanges. The differences among the three methodologies (visual inspection of sea surface topography maps, Okubo–Weiss criterion, and FTLE method) to identify and track eddies will be evaluated, and the impact of these differences on the estimates of the transport by the eddies will be investigated. Animations of time series of FTLE field have been made available for the reader's convenience

at http://www.rsmas.miami.edu/personal/fberon/lcs/nbc_dftle_bwd60d.avi (North Brazil Current retroflection region) and http://www.rsmas.miami.edu/personal/fberon/lcs/agu_dftle_bwd60d.avi (Agulhas Current retroflection region). Superimposed on the FTLE fields in these animations are selected sea surface height (gray contours) and regions where the Okubo–Weiss parameter equals two standard deviations in the whole domain (black contours), which is a commonly adopted threshold. These animations illustrate that the Okubo–Weiss criterion cannot detect as many coherent structures as the FTLE method.

Further Future Research Plans

I found particularly interesting the study of the underlying Lie–Poisson Hamiltonian structure of the fluid equations in their Eulerian form. The Hamiltonian formulation constitutes a unifying framework wherein symmetry properties are readily apparent and can be connected to conservation laws through Noether’s theorem. The existence of these conservation laws can then be used to derive stability theorems using Arnold’s method. The existence of a nonlinear (Lyapunov) stable state, in turn, allows for the establishment of nonlinear saturation bounds on the growth of perturbations to an unstable basic state in terms of the “distance” to the stable one. A common feature of these topics is that they combine challenging mathematical problems with direct relevance to practical questions of atmospheric and oceanic interest. For instance, nonlinear saturation bounds are potentially useful in the improvement of closures for eddy parameterizations in coarse-grained circulation models. Yet concrete applications are practically nonexistent, which leaves open an interesting avenue of investigation.

I am also interested in using Hamilton’s principle to derive approximate models. This technique has the advantage of enforcing the preservation of the symmetry-related invariants of the “primitive” model. I am particularly interested in deriving models with reduced vertical structure which allow for the incorporation of buoyancy fluxes through the ocean’s surface. The simplicity of the resulting model could be exploited to provide insights into the physics of the ocean’s mixed layer.

Finally, I find critically important the derivation of appropriate (geometric) numerical schemes that preserve the conservation laws of given set of fluid equations. This is an extremely challenging problem that has remained largely opened, although some progress has been recently reported in the literature.

2 Teaching Interests

My teaching experience as an undergraduate student includes two semesters of tutorials in a 2nd-year course for undergraduate students of engineering and physical oceanography at ITBA (Buenos Aires, Argentina) on classical mechanics (particle and systems of particles dynamics, rigid body dynamics, and Lagrangian and Hamiltonian dynamics). In the graduate program in physical oceanography at CICESE I was in charge for three semesters of the laboratory of the 2nd-year course on dynamical oceanography (Earth’s curvature effects, wind-driven ocean circulation models, geophysical waves, QG dynamics, equatorial dynamics, elements of turbulence). While being employed as a scientist at RSMAS (University of Miami) I taught a one semester course on scientific computing (floating-point arithmetic, root finding algorithms for nonlinear equations, numerical methods for ODEs and PDEs) for graduate students of mathematics and scientific computing at the Department of Computer Science of the University of Miami.

My teaching interests are varied. I would like to teach topics closely related to my research field, such as geophysical fluid dynamics, hydrodynamic stability, and Hamiltonian chaos. I would also like to teach basic tools of the dynamics of the continuous media or classical mechanics, such as asymptotics and variational calculus. I would also enjoy teaching introductory courses, such as fundamental calculus and mechanics, as well as dynamical and also descriptive oceanography.

Finally, I find very important to be actively involved in the formation of graduate students. I have already

served as a member of the committee of one PhD student, and I would very much enjoy to supervise graduate students (the various projects described above include salary support for graduate students).

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