Part II: Propagation of Inner Rainbands

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ABSTRACT

This is the second part of a study that examines spiral rainbands in a numerical simulation of Hurricane Bill (2009). This paper evaluates whether the propagation of inner rainbands in the Hurricane Bill simulation is consistent with previously proposed hypotheses. Results indicate that the propagation of inner rainbands is not consistent with gravity waves, vortex Rossby waves, or squall lines. An alternative hypothesis is offered, arguing that inner rainbands are simply convective clouds that are advected by the rapidly rotating tropical cyclone wind field while being deformed into spiral shapes. A summary and a discussion of the results of both Parts I and II are provided.

1. Introduction

Early radar observations of tropical cyclones (TCs) indicated that convection outside the eyewall was often organized into spirally banded structures rather than being uniformly distributed throughout the storms. Since then, spiral rainbands have been the topic of numerous studies. Some very early studies (e.g., Fletcher 1945; Wexler 1947) hypothesized that spiral rainbands formed from bands of clouds from the intertropical convergence zone (ITCZ) or cloud streets in the tropics that were coiled into TC circulations. Others argued that instabilities in the planetary boundary layers of TCs could result in the formation of spiral rainbands (e.g., Fung 1977).

Some radar observations indicated that some spiral rainbands remained stationary relative to the storm center (e.g., Allison et al. 1974; Fett and Brand 1975), while others moved around the center and also propagated radially outward (e.g., Senn and Hiser 1959). Inner spiral rainbands are an example of the latter type. They form just outside the eyewall and are often observed to propagate radially outward with time while moving around the storm center.

The focus of this paper is inner rainbands and their propagation mechanisms. Although not discussed by Houze [2010; see Fig. 1 in Moon and Nolan (2015), hereafter Part I], previous studies identified inner rainbands as those that form just outside the eyewall and propagate radially outward with time while being advected around the storm center (e.g., Chen and Yau 2001; Wang 2002a,b; Corbosiero et al. 2006, hereafter C06; Wang 2008, 2009). In particular, Wang (2008, 2009) proposed that inner rainbands are located within the rapid filamentation zone (RFZ; Rozoff et al. 2006)—a region of strain-dominated flow just outside the radius of maximum wind (RMW), which is characterized by a large negative radial gradient of angular velocity. In these regions, convection and its associated vorticity field are expected to be quickly filamented into thin strips, which are then lost ultimately to diffusion. RFZs typically cover annular regions from the RMW to 2 or 3 times the RMW. The description of secondary rainbands in Houze (2010) is very similar to that of inner rainbands provided in the aforementioned previous studies.

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Two hypotheses have been proposed so far to explain the radially outward propagation of inner rainbands. One hypothesis views inner rainbands as internal gravity waves that are generated from the rotating convective asymmetries in the eyewall (e.g., Kurihara 1976; Chow et al. 2002); the other hypothesis views them as vortex Rossby waves that emanate radially outward from the eyewall along a sharp negative radial gradient of potential vorticity (PV) just outside the RMW (e.g., Montgomery and Kallenbach 1997, hereafter MK97; Chen and Yau 2001; Wang 2002a,b; C06). The question as to how these inner rainbands propagate is of interest because the answer could lead to different implications for the impact of inner rainbands on TC intensity, which continues to be debated in the literature. If inner rainbands are characterized as vortex Rossby waves (e.g., anomalous positive PV bands), they could transport angular momentum radially inward through the asymmetrization process (e.g., Carr and Williams 1989; MK97; Nolan and Farrell 1999). This process increases the maximum tangential wind speed near the RMW, and therefore inner rainbands are perceived to have a positive impact on TC intensity. Previous studies with full physics numerical simulations of TCs have indicated that PV anomalies were generated within spiral rainbands (Chen and Yau 2001), especially in their stratiform regions (May and Holland 1999; Franklin et al. 2006). However, if inner rainbands propagate radially outward like gravity waves, then such gravity waves could conceivably have a negative impact on TC intensity by transporting angular momentum away from its core region. However, the magnitude of such angular momentum loss in the core has been shown to be negligibly small (e.g., Schecter 2008; Hendricks et al. 2010; Moon and Nolan 2010).

This is the second part of a study that examines spiral rainbands in a numerical simulation of Hurricane Bill (2009). The goal of this paper is to examine the mechanism by which inner rainbands propagate in the Hurricane Bill simulation described in Part I. This simulation uses higher resolution in comparison to previous studies so that spiral rainbands—including inner rainbands—are well represented. Model output from its innermost 1-km domain of 624 × 624 points produced every 2 min between 1800 UTC 19 August and 1800 UTC 20 August is used in this paper to examine the propagation of inner rainbands.

Here is how this paper is organized. Section 2 reviews two previously proposed hypotheses, and section 3 evaluates the propagation mechanism of inner rainbands and makes comparison with these hypotheses. Section 4 explores whether inner rainbands propagate like squall lines. Section 5 offers an alternative hypothesis for the propagation of inner rainbands. This paper concludes with a summary and discussion of the results of both Parts I and II in section 6.

2. A review of previously proposed hypotheses

a. Gravity waves

Since the first observations of radially outward-propagating inner rainbands, many previous studies (e.g., Anthes 1972; Kurihara and Tuleya 1974; Kurihara 1976; Diercks and Anthes 1976a,b; Chow et al. 2002) hypothesized that such inner rainbands bear the characteristics of outward-propagating internal gravity waves, which are generated by rotating convective asymmetries in the eyewall region. In this view, inner rainbands are regions of upward motion and condensation caused by outward-propagating gravity waves. Kurihara (1976) found in a linear stability analysis on a TC-like baroclinic vortex that the fastest-growing mode of outward-propagating gravity waves has the azimuthal wavenumber-2 (n = 2) structure with radial wavelength of 200 km. In addition to this outward-propagating mode, Kurihara (1976) also found two growing, inward-propagating modes.

There are a few variations in this hypothesis. Abdullah (1966) hypothesized that the breaking of radially inward-propagating gravity waves near the eyewall could induce squall lines that become radially outward-propagating inner rainbands. Willoughby (1977) treated gravity waves as linearized, nonhydrostatic perturbations to a TC-like barotropic vortex in gradient wind balance and found that the radial wavelength of the fastest-growing, outward-propagating gravity waves in Kurihara (1976) was too large to be supported in the considered model. In addition, the growth rate of the most unstable gravity waves at a shorter radial wavelength was not sufficient, and the lowest possible frequency of gravity waves excited in the eyewall region in the model was higher than that of inner rainbands deduced from observations. Willoughby (1977) concluded that radially outward-propagating gravity waves do not form a plausible model for radially outward-propagating inner rainbands in TCs. Instead, Willoughby (1978a,b) argued that radially inward-propagating gravity waves, which have a more realistic, lower frequency and amplify with decreasing radial distance from the storm center, match the characteristics of observed inner rainbands more consistently. Such inward-propagating gravity waves propagate against the mean cyclonic flow in the azimuthal direction, but the magnitude of their tangential propagation speed is slower than that of the mean tangential velocity, so they are advected slowly in the
downwind direction. Willoughby (1979) hypothesized that the excitation of such inward-propagating gravity waves could occur near the periphery of a TC from interacting with its environment.

**b. Vortex Rossby waves**

The idea that inner rainbands might propagate as Rossby-like waves was first postulated by MacDonald (1968), who made a qualitative analogy between inner rainbands and planetary Rossby waves. Guinn and Schubert (1993, hereafter GS93) presented more concrete evidence that inner rainbands might be the manifestation of Rossby-like wave dynamics. By using the shallow-water equations, GS93 hypothesized that inner rainbands could form either by axisymmetrizing PV anomalies in the eyewall and their subsequent filamentation or by straining out and merging of asymmetric PV sources outside the eyewall.

MK97 coined the term “vortex Rossby waves” (VRWs) and presented the properties of their propagation and interaction with the basic state. First, MK97 linearized both the two-dimensional incompressible flow equations and the asymmetric-balance shallow-water equations on an $f$ plane, in which surface gravity waves were filtered out (Shapiro and Montgomery 1993), about a stable, monopole circular vortex that represents the rapidly rotating TC core. MK97 then derived a dispersion relation for small, linearized perturbations by using the Wentzel–Kramers–Brillouin (WKB) approximation. In addition, MK97 showed from numerical experiments that VRWs propagated radially outward with time from the core region of the basic-state vortex along the negative PV gradient that exists outside its RMW (Fig. 1) and hypothesized that inner rainbands are the manifestation of VRWs. Möller and Montgomery (2000) extended the MK97 formulation of barotropic perturbations on a barotropic basic-state vortex by deriving a dispersion relation for baroclinic perturbations on a barotropic basic-state vortex. McWilliams et al. (2003) generalized the MK97 results by applying a multiparameter, asymptotic perturbation expansion.

Reasor et al. (2000) and C06 presented observational evidence supporting the existence of VRWs in TCs. Reasor et al. (2000) derived vertical vorticity fields from airborne dual-Doppler radar observations of Hurricane Olivia (1994) and found that the propagation of $n = 2$ vorticity perturbations in the eyewall of Hurricane Olivia was consistent with VRWs. C06 analyzed land-based radar observations of Hurricane Elena (1985) and noted that there was an inner rainband located to the southwest of the storm center that seemed to propagate radially outward with time (Fig. 2). This rainband feature was well captured by the $n = 2$ component of radar reflectivity. The time evolution of a cross section through this feature (Fig. 3) revealed that the radar reflectivity feature associated with the rainband clearly propagated radially outward with time, and its radial phase speed was found to be consistent with the theory of VRWs.

Evidence of VRWs in numerical simulations of TCs was presented by Chen and Yau (2001), Chen et al. (2003), and Wang (2002a,b). Chen and Yau (2001) performed a numerical simulation of Hurricane Andrew (1992). The innermost domain of the simulation had $232 \times 169$ points of 6-km horizontal grid spacing and 25 vertical levels. They found that inner rainbands were collocated with positive PV, positive vertical velocity, and high cloud water mixing ratio throughout the simulation (see their Fig. 4). They also computed the azimuthal and vertical phase speeds of PV anomalies associated with inner rainbands, and they were consistent with the VRW theory. Chen et al. (2003) performed
an empirical normal mode analysis (Brunet 1994) to separate the dynamical impact of VRWs from that of gravity waves. It was found that in the region where inner rainbands were active, VRWs were more dominant than gravity waves in terms of wave activity. Wang (2002a,b) performed a numerical simulation of a TC under idealized no-mean-flow conditions. The innermost domain of the simulation had \(108 \times 108\) points of 5-km horizontal grid spacing and 21 vertical levels. Wang (2002a,b) found that inner rainbands just outside the eyewall of the simulated TC vortex were collocated with higher simulated reflectivity values and positive PV anomalies and that the azimuthal phase speed of PV anomalies associated with the rainbands was consistent with the VRW theory [see Fig. 12 of Wang (2002b)].

3. Propagation of an inner rainband near 0900 UTC 20 August

Figure 4 shows horizontal cross sections at \(z = 3.2\) km of reflectivity every 4 min between 0900 and 0920 UTC 20 August from the Hurricane Bill simulation described in Part I. During this period, an inner rainband located to the southeast of the storm center appears to propagate radially outward with time. Overlaid on the horizontal cross sections as thick and thin black contours marking positive and negative parts of its \(n = 2\) reflectivity, respectively. The \(n = 2\) reflectivity appears to capture the radial outward propagation of this rainband. How this inner rainband propagates is now evaluated and compared to each of the above hypotheses.

a. Gravity waves

If inner rainbands propagate like gravity waves that are generated by rotating convective asymmetries and associated heat sources in the eyewall, they need to be collocated with positive vertical velocity anomalies that are typically associated with radially outward-propagating internal gravity waves (e.g., Holton 2004). Figure 5 shows horizontal cross sections at \(z = 3.2\) km of \(n = 2\) vertical velocity every 4 min between 0900 and 0920 UTC 20 August, with thick and thin black contours marking positive and negative parts of \(n = 2\) reflectivity at the same time as in Fig. 4. Examining the evolution of \(n = 2\) reflectivity and \(n = 2\) vertical velocity shows that they are not consistently collocated with each other. Comparing \(n = 2\) reflectivity
with $n = 2$ pressure anomalies at the same height during the same time also shows that they are not consistently collocated with each other (not shown), which suggests that this rainfall does not propagate like gravity waves. Sawada and Iwasaki (2010) also found that asymmetric pressure anomalies were not correlated with rainbands. This result is not surprising because the radial phase speed and radial width of outward-propagating gravity waves in previous studies are faster and larger than those associated with the observed inner rainbands.

b. Vortex Rossby waves

VRWs are PV bands that emanate radially outward from near the eyewall along the sharp negative PV gradient. If the propagation of inner rainbands has a physical connection to VRWs, there should be some clear correlations between PV bands and inner rainbands. Figure 6 shows horizontal cross sections at $z = 3.2 \text{ km}$ of $n = 2$ PV (color) and $n = 2$ reflectivity (solid black lines) during the same period as in Fig. 4. They indicate that the positive part of the $n = 2$ reflectivity associated with the rainband (thick solid black lines) is not collocated with the positive component of the $n = 2$ PV. Instead, a dipole-like structure of the $n = 2$ PV is collocated with the positive component of the $n = 2$ reflectivity during its radially outward propagation.

The main idea of the previous studies that argued inner rainbands propagate like VRWs is that inner rainbands are associated with positive PV anomalies. It is possible that PV fields from numerical simulations of TCs performed with 1-km horizontal grid spacing are too noisy and that some sort of smoothing is required to extract the PV signals associated with inner rainbands. To address this possibility, reflectivity and PV fields have been interpolated to a coarser 3-km horizontal grid, and the same analysis shown in Figs. 4–7 has been applied. However, the positive component of the $n = 2$ smoothed reflectivity associated with the rainband is still not collocated with the positive component of the $n = 2$ smoothed PV (not shown).

It is important to note that numerical simulations of TCs used in previous studies to evaluate the propagation of inner rainbands to the VRW theory used relatively coarse resolution [e.g., 6 km and 25 vertical levels in Chen and Yau (2001); 5 km and 21 vertical levels in Wang (2002a, b)]. At this resolution, the PV fields associated with inner rainbands do not show horizontal dipoles of PV [see Fig. 4 of Chen and Yau (2001) and Fig. 12 of Wang (2002b)]. We have repeated the Hurricane Bill simulation with the horizontal grid spacing of 5 km and 27 vertical levels, and the PV fields look very similar to those presented in the previous studies. Horizontal PV dipoles present in the 1-km simulation are no longer evident in the 5-km simulation (not shown). This suggests that numerical simulations of TCs should have the adequate resolution to resolve these finescale structures if they are to be used for comparing the propagation of finescale inner rainfall structures to the VRW theory.

Since inner rainbands are trailing features, examining the real part of the $n = 2$ Fourier decomposition coefficient—or taking a cross section through the $n = 2$ Fourier decomposition coefficient as done in C06—will show that these trailing rainbands propagate radially outward with time because of their apparent propagation along a fixed line. Figure 8 shows an idealized cross sections show that the inner band is associated with numerous dipoles of PV anomalies, with positive and negative PV anomalies on the radially outward and inward sides of the rainfall, respectively. Figure 7 shows horizontal cross sections at $z = 3.2 \text{ km}$ of $n = 2$ PV (color) and $n = 2$ reflectivity (solid black lines) during the same period as in Fig. 4. They indicate that the positive part of the $n = 2$ reflectivity associated with the rainband (thick solid black lines) is not collocated with the positive component of the $n = 2$ PV. Instead, a dipole-like structure of the $n = 2$ PV is collocated with the positive component of the $n = 2$ reflectivity during its radially outward propagation.

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FIG. 4. Horizontal cross sections at $z = 3.2$ km of reflectivity every 4 min between 0900 and 0920 UTC 20 Aug, with positive and negative parts of its $n = 2$ component shown as thick and thin solid black lines, respectively. Only $\pm 10$, $15$, $20$, and $25$-dBZ lines are shown. Dashed concentric circles are every 20 km.
trailing spiral band with end points at $r = 40$ and 60 km. This trailing feature is then translated only in the azimuthal direction with no radial movement at all. The position of the spiral band when it crosses $y = 0$ km (thick black line) is recorded. The inset in Fig. 8 shows the time evolution of the trailing spiral along $y = 0$ km (thick black line), and the trailing spiral appears to propagate radially outward with time, although, in fact,
it does not move in the radial direction at all. To determine whether trailing features like inner rainbands propagate radially outward with time, their intrinsic propagation, the absolute part of the $n = 2$ Fourier decomposition coefficient or its magnitude, must be examined, as shown in Fig. 11 of MK97 (reproduced here as Fig. 1).

Figure 9a shows the absolute part of the $n = 2$ Fourier decomposition coefficient of reflectivity at $z = 3.2 \text{ km}$.
Fig. 7. Horizontal cross sections at $z = 3.2$ km of $n = 2$ PV every 4 min between 0900 and 0920 UTC 20 Aug, with overlaid thick and thin solid black lines showing positive and negative parts of $n = 2$ reflectivity as in Fig. 4, respectively. Only $\pm 10\text{-}, 15\text{-}, 20\text{-},$ and $25\text{-} \text{dBZ}$ lines are shown. Dashed concentric circles are every 20 km.
during the same 20-min period. As shown before in Fig. 4, an inner rainband propagates radially outward, and the absolute part of the $n = 2$ reflectivity captures it well (Fig. 9a). The approximate slope of this radially outward-propagating reflectivity signal is marked by the black line in Fig. 9a. If this rainband propagates like VRWs, then it should be accompanied with coherent signals in the absolute part of the $n = 2$ PV. Figure 9b shows the absolute part of the $n = 2$ Fourier decomposition coefficient of PV at $z = 3.2$ km during the same 20-min period. Magenta lines in Fig. 9b show 14 dBZ of the absolute part of $n = 2$ reflectivity in Fig. 9a to help identify the location of the radially outward-propagating reflectivity signal; the black line is also copied from Fig. 9a. Figure 9b shows that there is some PV signal associated with the rainband, inside the magenta curve. However, the slope of this PV signal is different from that of the reflectivity signal associated with the rainband (black line in Fig. 9). Between 0900 and 0904 UTC, the reflectivity signal does not show much radial outward propagation, but the PV signal between $r = 45$ and 50 km is clearly pointed radially inward. Between 0906 and 0914 UTC, the slope of the PV signal is less than that of the reflectivity signal, as the PV signal is on the radially outward side of the black line near 0906 UTC but on its radially inward side by 0914 UTC. In addition, although the reflectivity signal associated with the rainband is the most dominant one in Fig. 9a, this PV signal is not the most dominant one. Figure 9b shows that stronger radially outward-propagating PV signals are found radially inward of the rainband, near $r = 28$ and 38 km at 0900 UTC. These PV signals are on the sharp negative PV gradient located just radially outward of the eyewall (Fig. 10). From examining Fig. 9a, it appears that the $n = 2$ reflectivity signal associated with the rainband moves from $r = 47$ km at 0904 UTC to $r = 53$ km at 0916 UTC, resulting in a radial speed of 6000 m/720 s = 8.3 m s$^{-1}$. The $n = 2$ PV signal associated with the rainband moves from
$r = 49$ km at 0904 UTC to $r = 51$ km at 0916 UTC, resulting in a radial speed of 2000 m/720 s = 2.8 m s$^{-1}$.

Figures 11 and 12 show the absolute parts of $n = 2$ Fourier decomposition coefficients at $z = 3.2$ km of reflectivity and PV between 0200 and 1800 UTC 20 August. The outward-propagating rainband examined in Fig. 9 can be found in the top panels of Fig. 11. During this period, there are many $n = 2$ reflectivity features propagating radially outward, especially near 1300 and 1730 UTC 20 August in Fig. 12. However, there are no coherent PV signals associated with these reflectivity features. Usually, it is either that reflectivity and PV signals are not at the same location or that they have different slopes (i.e., different radial speeds). In addition to outward-moving reflectivity features, there are some inward-moving reflectivity signals, such as near 0400 and 0500 UTC 20 August in Fig. 11. Figures 13 and 14 show the absolute parts of $n = 2$ decomposition coefficients of reflectivity and PV during the same period, but at $z = 2.0$ km. Again, there are many $n = 2$ reflectivity features that propagate radially outward, especially near 0900 UTC on Fig. 13 and 1230 and 1730 UTC 20 August on Fig. 14. However, there are no coherent $n = 2$ PV signals colocated with these $n = 2$ reflectivity features. Correlation between $n = 2$ reflectivity and $n = 2$ PV does not improve at $z = 1.0$ km or between 1800 UTC 19 August and 0200 UTC 20 August (not shown).

It is likely that $n = 1$ decomposition coefficients contain substantial power. However, it is difficult to separate the projection into $n = 1$ that is generated internally from that generated by the influence of environmental asymmetries. Previous studies might have focused on $n = 2$ or higher mostly for this reason. Comparison of $n = 3$ reflectivity and $n = 3$ PV at $z = 3.2$ km does not show coherent signals colocated in the same radius–time space (not shown).

In addition to the rainband shown in Fig. 4, numerous inner rainbands are present between 0200 and 1800 UTC 20 August. For example, between 1308 and 1328 UTC 20 August, there is an inner rainband located to the southeast of the storm center that appears to propagate radially outward with time (Fig. 15). However, unlike the inner rainband shown in Fig. 4, this inner rainband is not even captured by the positive part of its $n = 2$ component at all. Horizontal cross sections of PV (and its $n = 2$) at $z = 3.2$ km during the same period show that this rainband is associated with horizontal dipoles of PV anomalies (not shown).

4. Do inner rainbands propagate like squall lines?

Although their main focus was not on inner rainbands, many previous studies noted similarities between other types of rainbands in TCs and squall lines (e.g., Barnes et al. 1983; Powell 1990a,b) and argued that surface cold pools and mechanical lifting associated with them could be important for rainbands (e.g., Robe and Emanuel 2001; Didlake and Houze 2009; Sawada and Iwasaki 2010).

Figure 16 shows horizontal cross sections of virtual potential temperature at $z = 0.12$ km and vertical velocity at $z = 1.3$ km during the same period as in Fig. 4. Overlaid black lines in Fig. 15 show 30-dBZ reflectivity at $z = 3.2$ km to help identify the location of the inner rainband. These cross sections indicate that the inner rainband is hardly associated with surface cold pools and that the rainband is associated with some dipole-like structures of vertical velocity. It appears unlikely that surface cold pools and lifting associated with them play an important role for the propagation of this inner rainband. Horizontal cross sections of virtual potential temperature at $z = 0.12$ km and vertical velocity at $z = 1.3$ km for the rainband in Fig. 15 indicate the same results (not shown).

5. An alternative hypothesis for the propagation of inner rainbands

The propagation of the inner rainband in Fig. 4 has been compared to the previously proposed hypotheses, and the propagation mechanism is not explained well by them. Then how does this rainband propagate? Figure 17 shows horizontal cross sections at $z = 3.2$ km of reflectivity every 6 min from 0830 to 0912 UTC 20 August. Backtracking the inner rainband previously shown in Fig. 4 suggests that convective clouds that become the middle and downwind parts of the rainband between 0900 and 0920 UTC (as marked by dashed magenta circles in Fig. 17) appear to originate upstream from near the eyewall ($r = 40$ km). However, following convective clouds that become the upwind end of the same rainband (as marked by solid magenta circles in Fig. 17) indicates that these clouds are mostly advected in the azimuthal direction (near $r = 70$ km) and stay apart from the eyewall. Convective clouds that become the rainband after 0900 UTC are not located next to each other at earlier times. Instead, this inner rainband appears to “propagate” radially outward with time between 0900 and 0920 UTC because the rainband is simply growing by the convergence of convective clouds of different origins.

An alternative hypothesis is offered that involves the vertical wind shear–induced indirect secondary circulation in the upshear region and the deformation of the low-level horizontal wind field. To help illustrate the processes that are responsible for the movement of
FIG. 11. Absolute parts of $n = 2$ Fourier decomposition coefficients at $z = 3.2$ km of (left) reflectivity and (right) PV between 0200 and 1000 UTC 20 Aug.
Fig. 12. As in Fig. 11, except between 1000 and 1800 UTC 20 Aug.
FIG. 13. As in Fig. 11, except at \( z = 2.0 \) km.
FIG. 14. As in Fig. 12, except at $z = 2.0$ km.
the inner rainband, a 2-h composite of 2-min model output between 0800 and 1000 UTC 20 August is created. Figure 18 shows horizontal cross sections of storm-relative radial velocity averaged between $z = 8.3$ and $10.9 \text{ km}$, vertical velocity averaged between $z = 3.2$ and $6.1 \text{ km}$, and storm-relative radial velocity averaged between $z = 0.5$ and $2.0 \text{ km}$ from the 2-h composite. The 850–200-hPa vertical wind shear vector is from the west.
during this period. Many previous studies have shown that rising (sinking) motions are favored in the downshear (upshear) regions because of the upward-sloped isentropes into the direction of the vertical wind shear vector and adiabatic flows along these isentropes (e.g., Jones 1995; Frank and Ritchie 2001). Figure 18 shows that there is a vertical wind shear–induced indirect secondary circulation in the upshear region (i.e., to the west) that is made of upper-level radial inflow and low-level radial outflow, with sinking motions between them (e.g., Wong and Chan 2004; Xu and Wang 2013). A recent observational study by DeHart et al. (2014) also shows this indirect secondary circulation in the upshear region. This indirect secondary circulation rotates anticyclonically with height, consistent with the vertically decaying cyclonic tangential wind field.

Exchanging the right column of Fig. 16 (see also Fig. 20b) shows that small-scale downdrafts in the upshear region near the eyewall are quite strong (e.g., \( w < -5 \text{ ms}^{-1} \)) and transient. Close examination of individual 2-min outputs between 0800 and 1000 UTC on August 20 also supports the same characterization (not shown). Once these strong downdrafts reach near the surface, air transported downward with the downdrafts will spread out horizontally. Since there are many such strong downdrafts, some of downward-transported air could interact, leading to horizontal convergence in some locations, which could be strong enough to initiate convection. In this region, equivalent potential temperature decreases with increasing height from the surface to near \( z = 3.5 \text{ km} \) so that convection is favored if sufficient forcing exists (see Fig. 21e).

Let us assume for a moment that convection forms in the upshear region. Figure 19a shows the horizontal wind field averaged between \( z = 0.5 \) and \( 2 \text{ km} \) from the 2-h composite. Any convection formed in this region will be advected radially outward (Fig. 18c) and also into the cyclonic direction by the low-level tangential wind field. This 0.5–2.0-km layer is chosen because inner rainbands are made of multiple convective clouds interacting, leading to horizontal convergence in some locations, which could be strong enough to initiate convection. In this region, equivalent potential temperature decreases with increasing height from the surface to near \( z = 3.5 \text{ km} \) so that convection is favored if sufficient forcing exists (see Fig. 21e).

The eigenvalues indicate the magnitude of the deformation field, while the eigenvectors show its direction. Figure 19b shows that the deformation field induced by the differential rotation associated with the TC circulation is substantial near the eyewall (~10\(^{-3}\) \text{s}^{-1}) such that any convective features generated in this region will be deformed into spiral shapes over time. To illustrate how strong this deformation field is, a simple two-dimensional flow equation for a tracer \( C \),

\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = 0, \tag{2}
\]

is solved by using the above depth-averaged, time-averaged horizontal wind field as \( U \) and \( V \) in Eq. (2). The computational domain is a 250-km square box of 1-km grid. A fifth-order upwind scheme and the fourth-order Runge–Kutta method are used to integrate Eq. (2) forward in time for 30 min with a time step of 1.0 s. At \( t = 0 \text{ min} \), a localized tracer source with a Gaussian bubble-like structure, \( C = 1.0 \times \exp \left(- \left(\frac{x-x_{\text{center}}}{5000}\right)^2 - \left(\frac{y-y_{\text{center}}}{5000}\right)^2\right) \), (3)

is initialized at \( x_{\text{center}} = -25 \text{ km} \) and \( y_{\text{center}} = -25 \text{ km} \), which is the approximate location where the strong convective-scale downdrafts reach near the surface (see Figs. 16 and 18). Figure 20 shows the time evolution of this tracer every 5 min. It shows that this tracer initially at \( x = -25 \text{ km} \) and \( y = -25 \text{ km} \) moves radially outward and also into the cyclonic direction over time. The radial outward end of this tracer is near \( r = 42 \text{ km} \) at \( t = 0 \text{ min} \) but \( r = 52 \text{ km} \) after \( t = 15 \text{ min} \). At the same time, the deformation field associated with the mean horizontal wind field (Fig. 19b) turns the initially circular tracer into spiral shapes.

The two-dimensional tracer equation [Eq. (2)] is integrated forward for 40 min, but with different initial conditions consistent with Fig. 17, which shows that the inner rainbands are made of multiple convective clouds originating from different places. In Fig. 21a, there are two Gaussian blobs of tracer at \( t = 0 \text{ min} \): one is centered at \( x = -35.0 \text{ km} \) and \( y = 0.0 \text{ km} \), and the other is centered at \( x = -43.5 \text{ km} \) and \( y = -11.6 \text{ km} \). The tracers in Fig. 21b are centered at \( x = -40.0 \text{ km} \) and \( y = 0.0 \text{ km} \), and at \( x = -51.5 \text{ km} \) and \( y = 0.0 \text{ km} \). Figure 21 shows the time evolution of these tracers every 10 min. In Fig. 21a, both tracers move radially outward and into the cyclonic direction while being deformed into spiral shapes. However, because the radial outflow is stronger near the eyewall, the inner tracer catches up the outer tracer by \( t = 20 \text{ min} \), and they become attached to form a single spiral banded tracer at later times. Figure 21b shows a similar evolution that these two tracers become a single band, although they are less coherent as a single.

\[
\begin{align*}
\mathbf{V} & = \begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}
\end{bmatrix},
\end{align*}
\tag{1}
\]
The evolution of tracers in Figs. 20 and 21 looks qualitatively similar to the evolution of the inner rainband examined in Figs. 4, 15, and 17.

To summarize, our alternative hypothesis proposes that inner rainbands propagate radially outward with time because convective features (and hydrometeors associated with them) generated in the upshear region are simply advected by the swirling mean horizontal wind field, while being deformed into spiral shapes by its deformation field. The presence of significant
horizontal deformation just outside the eyewall has been noted previously in Rozoff et al. (2006) and Wang (2008) in their study of moat formation. Convection in the upshear region could be triggered from horizontal convergence created episodically by strong, small-scale downdrafts in the region that spread out horizontally after they reach the surface. These strong downdrafts are part of the indirect overturning secondary circulation that develops in response to the presence of environmental vertical wind shear. These inner rainbands are not able to establish coupling to the boundary layer through surface cold pools, so
they will decay quickly, leading to their transient characteristics.

It should be emphasized that the low-level radial outflow in the upshear region, which is part of the indirect secondary circulation induced by the vertical wind shear, plays an important role for the radial outward movement of these spiral tracers and inner rainbands. To illustrate this, the two-dimensional tracer equation [Eq. (2)] is integrated with the same initial condition as Fig. 20, except that the horizontal wind field now has only the tangential component of Fig. 19a. Figure 22 shows that the tracer moves in the cyclonic direction and

Fig. 17. Horizontal cross sections at $z = 3.2$ km of reflectivity every 6 min from 0830 to 0912 UTC 20 Aug. Dashed concentric circles are every 20 km. Solid and dashed magenta circles track the upwind and middle parts of the rainband, respectively.
becomes deformed into spiral shapes. However, unlike in Fig. 20 it does not show any significant movement in the radial direction.

In the absence of diabatic heating and friction, a Gaussian PV/vorticity bubble (of positive sign only) in the two-dimensional flow would behave somewhat like the passive tracer described in this section, except that it may also experience some radially outward propagation (e.g., MK97; Möller and Montgomery 1999). However, it is difficult to expect the TC environment to be free of diabatic heating, as some forms of convective activities are almost always present, especially near the eyewall. Plus, the dominant form of PV anomalies generated by

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**FIG. 18.** Horizontal cross sections of (a) storm-relative radial velocity averaged between $z = 8.3$ and 10.9 km, (b) vertical velocity averaged between $z = 3.2$ and 6.1 km, and (c) storm-relative radial velocity averaged between $z = 0.5$ and 2.0 km from a 2-h composite of 2-min data between 0800 and 1000 UTC 20 Aug. Zero lines are shown as thick black lines in (a) and (c). Dashed concentric circles are every 20 km.

**FIG. 19.** (a) Horizontal wind speed (color) and vectors averaged between $z = 0.5$ and 2.0 km from a 2-h composite of 2-min data between 0800 and 1000 UTC 20 Aug. (b) Deformation magnitude (color) and vectors of the depth-averaged, time-averaged horizontal wind field shown in (a).
diabatic heating will have dipolar structures in both horizontal and vertical directions because of the horizontal and vertical gradients of diabatic heating (e.g., Nolan et al. 2007) as well as by the tilting of horizontal vorticity lines into the vertical direction by convective-scale motions in the TC wind field environment (e.g., Powell 1990a).

6. Summary and conclusions

This two-part study has examined spiral rainbands in a 3-day numerical simulation of Hurricane Bill (2009). The simulated storm track and intensity fluctuations were in excellent agreement with the observed values. This simulation was performed with higher resolution compared to previous studies so that spiral rainbands were better resolved.

Part I of this study has evaluated the structures of spiral rainbands in the simulation and found that the structures of rainbands in the simulation were in good agreement with previous observations. Four types of rainbands have been identified: principal, secondary, distant, and inner rainbands. Principal and secondary rainbands are tilted radially outward with height, while distant rainbands are tilted radially inward with height. Principal rainbands were relatively stationary and had a strong azimuthal wavenumber-1 signature, while secondary rainbands were more transient. Principal rainbands were bounded by very dry air to their radially outward sides. Both principal and secondary rainbands had an overturning secondary circulation and enhanced tangential velocity on their radially outward sides, which are the typical kinematic features associated with rainbands in previous observations. Distant rainbands had locally dense surface cold pools. Inner rainbands were shallow convective features that appeared to originate from near the eyewall.

This paper, the second part of the study, has investigated how inner rainbands that formed near the eyewall propagated radially outward with time in the Hurricane Bill simulation and compared against previously proposed
hypotheses. This paper found that their propagation was not consistent with gravity waves because inner rainbands were not consistently collocated with vertical velocity (or pressure) signals expected if these rainbands propagated like gravity waves. The propagation of inner rainbands was not consistent with VRWs because inner rainbands were associated with horizontal dipoles of PV anomalies, not positive PV anomalies; and their radial outward propagation was not consistently collocated with PV signals expected from the VRW theory in the radius–time space of the Fourier decomposition coefficients. In addition, inner rainbands did not have dense surface cold pools that are typically associated with squall lines, so it was unlikely that inner rainbands propagated like squall lines.

An alternative hypothesis was offered—that inner rainbands are simply convective clouds (and hydrometeors associated with them) that are advected by the rapidly rotating TC wind field while being deformed into spiral shapes. Advecting localized tracer sources in a simple two-dimensional flow equation alone reproduced the qualitative characteristics of inner rainband propagation. The radial outward movement of tracers (and inner rainbands) is due to the presence of the low-level radial outflow, which is part of the vertical wind shear–induced, indirectly overturning secondary circulation in the upshear region. The strong deformation field associated with the swirling TC wind field is responsible for spiral shapes of tracers.

This study has examined mostly the structures and movements of spiral rainbands in a numerical simulation of Hurricane Bill (2009). However, there are many questions about these spiral rainbands that have not been addressed, such as their formation mechanisms. Willoughby et al. (1984) hypothesized that principal rainbands mark the boundary between TCs and their surrounding environment and that these principal rainbands form through the interaction between them. However, as noted in Houze (2010), how this interaction occurs is not fully quantified. Recently, Riemer and Montgomery (2011) and Akter and Tsuboki (2012) examined this process. Whether their results also apply for principal rainbands in the Hurricane Bill simulation will be explored in a future study. The propagation and formation mechanisms of secondary rainbands should also be examined.

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REFERENCES


