Understanding internal wave-wave interaction patterns observed in satellite images of the Mid-Atlantic Bight

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Abstract:

Many internal wave-wave interaction patterns have been observed in satellite images. However, very few studies have been made on understanding these patterns. Internal wave interactions may result in exceptionally large amplitudes in the interaction zone, which in turn pose threats to underwater structures. In this paper, we analyze the characteristics of interaction patterns observed in satellite images of the Mid-Atlantic Bight, such as internal wave phase shifts and amplitude changes. Based on these characteristics, we categorize the patterns into four different types: Mach interaction; regular interaction with prominent positive phase shifts and an amplitude decrease in the interaction zone; regular interaction with prominent negative phase shifts and an amplitude increase in the interaction zone; wave interactions without phase shifts.

We provide a detailed analysis of one observed interaction pattern within each category, and compare our findings with existing analytical and numerical models for two-soliton interaction. One important result from this study of interaction patterns is that the patterns alone can be used to deduce how the amplitude changes in the potentially hazardous interaction zone. This paper
thus proves that high resolution satellite images can provide a useful and efficient means of studying internal wave interaction.

**Index Terms:** Internal wave-wave interaction; phase shifts; amplitude changes; satellite images; Mid-Atlantic Bight.

1. **Introduction**

Internal wave packets propagating in the ocean often interact with each other. The resulting internal wave-wave interaction pattern is a common and interesting feature observed frequently in satellite images, both synthetic aperture radar (SAR) and optical images in partly cloud-free condition. When internal wave trains have propagation directions that are very close to each other, they often merge to form an extended packet. When their propagation directions are not very close to each other, some interaction patterns show prominent phase shifts, while some other patterns show no phase shifts and indicate that interactions have very small effects on internal wave packets. Studying these interaction patterns is not a trivial task, not only because of the complex dynamics involved but also because of very few available geophysical observations. For example, ship radar images have been shown to be useful to follow one internal wave packet and study its evolution [1]; however, in the case of an internal wave interaction, the radar’s small coverage makes it difficult to observe the complete interaction pattern. In-situ data are expensive and limited to point measurements, which makes it impossible to catch the phase shifts due to an interaction. As a result, satellite imagery is the best and most cost-effective data source to observe and study internal wave interaction patterns. A few satellite images show patterns of
more than two interacting wave packets. In this paper, for simplicity, we will focus on the interactions of two internal wave packets only.

In satellite images, we observe many internal wave interaction patterns. However, only very few studies have been made on understanding the internal wave dynamics behind these patterns. Hsu et al. [2] observed the nonlinear internal wave interaction in a RADARSAT ScanSAR Wide image, but they did not focus on the interaction characteristics. Chen et al. [3] used the theory of resonant interaction of two internal wave packets based on the Kadomtsev-Petviashvili (KP) equation to explain a satellite image exhibiting special patterns, providing a mechanism for boosting internal wave amplitudes. Xue et al. [4] investigated qualitatively the phase shifts and amplitude changes associated with regular interaction patterns visible in satellite images using the KP model.

During the Nonlinear Internal Wave Initiative/Shallow Water 2006 (NLIWI/SW’06) experiment, 14 interaction patterns were observed in satellite images of the Mid-Atlantic Bight (MAB). We categorize them into four different types based on the characteristics the patterns exhibit: Mach interaction (1/14); regular interaction with prominent positive phase shifts and an amplitude decrease in the interaction zone (6/14); regular interaction with prominent negative phase shifts and an amplitude increase in the interaction zone (4/14); wave interactions without phase shifts (3/14). In this paper, we analyze one interaction pattern from each category in detail. The goal is to determine the patterns’ characteristics, compare them with results obtained from existing two-soliton interaction models, and identify means of relating the observed patterns to the interior ocean dynamics associated with the interactions.
This paper is organized as follows: In section II, existing theoretical studies of solitary wave interactions are presented. In section III, we give a brief description of the SW’06 experiment and satellite observations used in this study. Section IV shows and analyzes four different interaction patterns we observe and compares them with the existing analytical and numerical studies. Conclusions are presented in section V.

2. Theoretical background

Most observed internal waves are first baroclinic mode waves (referred to as mode-1 internal waves). All isopycnals are displaced in the same direction in the mode-1 structure. Mode-1 internal waves can be treated as solitary-like, thus internal wave interactions can be studied as solitary wave interactions. Miles [5]-[6] presented a comprehensive theoretical study of small-amplitude shallow water solitary wave interactions. Generally, one can distinguish between three broad types of interaction: Mach interaction, regular interaction, and resonant interaction. In the following, we introduce the relevant interaction variables and summarize Miles’ criteria of distinction. In a two-layer stratified water, assume that the pre-interaction amplitudes of two long internal waves, normalized by the upper layer depth, are $\eta_1$ and $\eta_2$, and both are much smaller than 1.

Mach interaction is to be expected when

$$\psi_+ < \sin^2 \frac{\psi}{2} < \psi_+.$$  \hspace{1cm} (1)

where $\psi$ is the pre-interaction angle between the two internal wave propagation directions, and

$$\psi_\pm = \frac{3}{4} \left( \sqrt{\eta_1} \mp \sqrt{\eta_2} \right)^2.$$ For interactions where $\eta_1 = \eta_2$, the post-interaction angle is given by
\( \psi_p = 2\sqrt{3\eta_{1,2}} \). It is only dependent on the incoming wave amplitudes and is usually much larger than the pre-interaction angle \( \psi \). The length of the Mach stem, the merged front in the interaction zone, linearly increases with time. The post-interaction amplitudes, given by

\[ \eta_{1p,2p} = \left( \frac{\psi}{2\sqrt{3\eta_{1,2}}} \right)^2 \eta_{1,2}, \]

are much smaller than those before the interaction. Here \( \psi \) is used in its radian value. Note that we use a subscript “p” to denote post-interaction variables.

Regular interaction is to be expected when

\[ \sin^2 \frac{\psi}{2} < \psi_- \; \text{or} \; \sin^2 \frac{\psi}{2} > \psi_+, \]  

(2)

For regular interactions, the post-interaction wave crests are parallel to the pre-interaction wave crests (\( \psi = \psi_p \)), and the amplitudes of the post-interaction waves are the same as the pre-interaction waves’ (\( \eta_{1,2} = \eta_{1p,2p} \)).

Resonant interaction should occur when

\[ \sin^2 \frac{\psi}{2} = \psi_- \; \text{or} \; \sin^2 \frac{\psi}{2} = \psi_+. \]  

(3)

The merged front grows infinitely and no post-interaction waves can be seen. This is essentially a three-wave interaction. In the following, we will disregard this type and focus on two-soliton interactions.

In the real world, the normalized internal wave amplitudes sometimes do not satisfy Miles’ small amplitude requirement. Tanaka [7] examined large amplitude wave interactions with \( \eta_{1,2} = 0.3 \) by numerical simulation. The critical \( \psi \) for Mach interaction is 75°, which is much smaller than Miles’ prediction of 108° for this amplitude. Regular interaction happens for any \( \psi \) larger than the critical value. We expect that the critical \( \psi \) for Mach interaction decreases
with a larger wave amplitude. Note that Tanaka studied reflection of an obliquely incident solitary wave by a vertical wall, so that the incidence angle and the reflection angle in his paper are half of $\psi$ and $\psi_p$ used here, respectively. Another difference between the two studies is that Tanaka’s $\psi_p$ decreases with $\psi$, while Miles’ theory gives a constant angle. Also, Tanaka’s $\eta_{1p,2p}$ are smaller than $\eta_{1,2}$, but larger than in Miles’ theory. Finally, Tanaka’s step angle ($\psi_s$), which is determined from a pair of imaginary lines drawn along the edges of the merged fronts, is used to describe the merged front’s growth rate. It is smaller than Miles’ prediction. Tanaka concludes that large amplitudes tend to prevent the Mach interaction from happening. Even when Mach interaction happens, it has characteristics of both a Mach and a regular interaction when compared with Miles’ theory. The first quantitative study of oblique interactions of internal waves was carried out by Wang and Pawlowicz [8]. They used time sequences of photogrammetrically rectified oblique images obtained from a circling aircraft and simultaneous water column data of the wave structure to study internal wave interactions. The normalized amplitudes in their study area are greater than 0.75. According to that study, the likelihood of Mach interaction for waves with large amplitudes is overestimated by Miles’ theory. In other words, the patterns satisfying Miles’ Mach interaction criteria were observed to exhibit the characteristics of regular interactions. Their findings thus confirm Tanaka’s numerical results. Therefore, if an observed interaction pattern satisfies Miles’ Mach interaction criteria, but the post-interaction angle and wave amplitudes are very close to the pre-interaction ones, we treat the pattern as a regular interaction.
The Korteweg-de Vries (KdV) equation is often used to describe internal waves. However, for two obliquely interacting solitons, the 2-D KdV equation describing nonlinear dispersive internal waves, known as the Kadomtsev-Petviashvili (KP) equation, is more appropriate. Peterson and van Groesen [9]-[10] have explicitly studied the two-soliton solution by decomposing it into a sum of two single solitons and an interacting soliton using the KP equation. Peterson et al. [11] used the KP model to study shallow water soliton interaction and suggested it as a possible model for extreme waves. Their paper provides a detailed analysis of the phase shifts and amplitude change in the interaction zone due to the interaction. Also, they provide an analytical two-soliton solution, which is a stable-stage solution for an existing interaction area. In the following, we will make use of it for studying regular interaction patterns in satellite images.

The KP equation describing nonlinear dispersive internal wave is given by:

\[(\eta_t + c_0 \eta_x + c_1 \eta \eta_x + c_2 \eta_{xxx})_x + \frac{c_0}{2} \eta_{yy} = 0\]  \hspace{1cm} (4)

Its analytical solution is usually constructed on the basis of the canonical form:

\[(u_t - 6uu_x + u)_x + 3u_{yy} = 0\]  \hspace{1cm} (5)

The two equations can be converted into each other through a coordinate transform.

In the following, we focus on (5) for its generality. The two-soliton solution for (5) reads

\[u(x, y, t) = 2 \frac{\partial^2}{\partial x^2} \ln \theta\]  \hspace{1cm} (6)

where \(\theta = 1 + e^{\varphi_1} + e^{\varphi_2} + A_{12} e^{\varphi_1} e^{\varphi_2}, \varphi_{1,2} = k_{1,2} x + l_{1,2} y + \omega_{1,2} t\) are phase variables, and \(\kappa_1 = (k_1, l_1), \kappa_2 = (k_2, l_2)\) are the wave vectors of the incoming solitons. The “frequency” \(\omega_{1,2}\) can be found from the dispersion relation of the linearized KP equation:

\[P(k_{1,2}, l_{1,2}, \omega_{1,2}) = k_{1,2} \omega_{1,2} + k_{1,2}^4 + 3l_{1,2}^2 = 0\]  \hspace{1cm} (7)
Upon interaction, the two solitons undergo a phase shift: $\Delta_{12} = -\ln A_{12}$. Without losing generality, we assume $t=0$, then

$$ A_{12} = \frac{P(k_1-k_2, l_1-l_2, \omega_1-\omega_2)}{P(k_1+k_2, l_1+l_2, \omega_1+\omega_2)} = \frac{\lambda^2-(k_1-k_2)^2}{\lambda^2-(k_1+k_2)^2} \tag{8} $$

where $\lambda = \frac{l_1}{k_1} - \frac{l_2}{k_2}$. For a nonzero interaction angle, the interaction may have positive or negative phase shifts (if $A_{12} > 1$ and $0 < A_{12} < 1$, respectively). The amplitudes of the two incoming solitons are given by $u_{1,2} = 1/2k_{1,2}^2$. The two-soliton solution can be decomposed into a sum of two incoming solitons $s_1, s_2$ and the interaction soliton $s_{12}$ [9]-[10]:

$$ u = s_1 + s_2 + s_{12} \tag{9} $$

when the counterparts $s_1, s_2$ and $s_{12}$ are defined as

$$ s_{1,2} = A_{12}^{1/2} k_{1,2}^2 \cosh(\varphi_{1,2} + \ln A_{12}^{1/2}) \tag{10} $$

$$ s_{12} = \frac{(k_1-k_2)^2 + A_{12}(k_1+k_2)^2}{2\Theta} \tag{11} $$

with $\Theta = \cosh[(\varphi_1 - \varphi_2)/2] + A_{12}^{1/2} \cosh[(\varphi_1 + \varphi_2 + \ln A_{12})/2]$

Regular interactions described by the KP equation have two categories: (1) Negative phase shifts when $0 < A_{12} < 1$, which results in an amplitude increase in the interaction zone. It typically occurs for interactions between solitons with comparable amplitudes. (2) Positive phase shifts when $A_{12} > 1$, which results in an amplitude decrease in the interaction zone. It typically occurs for interactions between solitons with large amplitude differences. The two categories show different patterns. The first one has a merged front in the interaction zone and thus looks similar to Mach interaction. The second one differs in that it does not have a merged front in the interaction zone. In this paper, we compare the characteristics of the observed Mach interaction.
pattern with Tanaka’s findings and simulate the other three observed interaction patterns using the KP model.

3. Satellite observations

The SW’06 experiment took place on the continental shelf off the coast of New Jersey. Many satellite images, including Envisat image mode ASAR, ERS-2 SAR, RADARSAT-1 standard mode SAR, SPOT 2/4 high resolution visible panchromatic (HRV-PAN) images, were received and processed at the University of Miami’s Center for Southeastern Tropical Advanced Remote Sensing (CSTARS). All of the SAR images have a resolution of 25 m, and the SPOT HRV-PAN images have a resolution of 10 m. The orbital currents induced by internal waves can modulate sea surface capillary and short gravity waves into smooth divergent and rough convergent zones. Internal waves become visible because the radar backscattering properties of the sea surface change with the roughness [12]. The modulation depths of internal wave signatures in SAR images are stronger and weaker backscatter relative to the background corresponding to the rougher and smoother areas on the sea surface. Many factors including radar parameters, wind conditions, and internal wave related parameters affect the modulation depths of internal wave signatures in the images [13]-[15]. For an interaction pattern in a satellite image, we assume that all the factors except for internal wave amplitude are the same for the whole pattern. Consequently, if the modulation depths of internal waves are larger in the interaction zone, it indicates the amplitudes are larger there. Oppositely, if the modulation depths of internal waves are smaller in the interaction zone, it indicates smaller amplitudes there [13]-[15]. In optical
images, internal waves become visible because the sunglitter radiance depends on the surface roughness and slope distribution as well. In the non-sunglint area, the spatially varying roughness modulates the diffuse reflection of sunlight [16]-[18]. Essentially, the signatures of internal waves in optical images are related to the surface roughness as well so that all discussed above for SAR images also applies to the interaction patterns observed in optical images.

The internal wave interaction signatures in satellite images are sometimes too vague to study due to high wind speeds or other atmospheric effects. However, we observed 14 clear two internal wave packets interaction patterns in the satellite images from the SW’06 experiment. In this paper, we analyze one interaction pattern from each category. Figure 1 shows the locations of four interaction patterns cut from the original images over the bathymetry. The rectangle (a) outlines the location of the case of Mach interaction shown in Figure 2; The rectangle (b) the case of regular interaction with prominent positive phase shifts and an amplitude decrease in the interaction zone shown in Figure 5; The rectangle (c) the case of regular interaction with prominent negative phase shifts and an amplitude increase in the interaction zone shown in Figure 7; The rectangle (d) the case of interaction without phase shifts shown in Figure 11.

4. Internal wave interaction patterns

4.1 Mach interaction

We can see two internal wave packets propagating in different directions in Figure 2. When such two packets interact with each other for a period of time, merged fronts are generated in the interaction zone. This results in an interlocked, zipper-like interaction pattern observed in the SPOT-4 HRV-PAN image taken on August 05, 2006. Although satellite images are snapshots
only, the information from the spatial domain can be transferred to the temporal domain of the interaction process. Intuitively, the pattern implies the first waves in both packets interact first, and then the second waves in both packets interact. Starting from the third waves, interactions become more complicated. The post-interaction wave from the second wave-wave interaction interacts with the third wave in the right packet, and then the resulting post-interaction wave interacts with the third wave in the left packet. Here we make the simplification that the third waves in both packets directly interact with each other. Moreover, since the interactions in the rear are not as strong as in the front, we only focus on the interactions of the first three waves in both packets.

Figure 3(a) shows a sketch of the interaction pattern of the first three waves in both packets. Packet A stands for the wave packet on the left and packet B stands for the wave packet on the right. The pre-interaction waves in packet A are marked as A1, A2, and A3 orderly and the corresponding post-interaction waves are labeled a1, a2, and a3. In the same way, the pre-interaction waves in packet B are marked as B1, B2, and B3 orderly and the corresponding post-interaction waves are labeled b1, b2, and b3. The merged front generated due to the interaction of A1 and B1 is marked as AB1. The interaction sketch of the first waves is shown in Figure 3(b). The angles between A1 and AB1, B1 and AB1 are approximately 27° and 18°, respectively. The interaction angle between packets A and B is approximately 45°. Wave crests after the interaction are not parallel to their pre-interaction orientations. The angle between a1 and AB1 is approximately 38°, and the angle between b1 and AB1 is approximately 40°. The post-interaction angle between a1 and b1 is approximately 78°. The lengths of the first three
merged fronts are 2.80 km, 1.43 km, and 0.98 km. The length increases with time, which is an unsteady interaction, indicating a Mach interaction ([5]-[6], [8], [19]). The merged front in a Mach interaction is named Mach stem. Assuming the amplitude difference of the first and the second waves in the packet is very small, we can retrieve the temporal development of the first Mach stem from the spatial snapshot of the first two Mach stems. This is accomplished by connecting the edge points of the first two Mach stems on both sides and extending the lines backward, yielding the cross point, which is estimated to be the generation location of the Mach stems (see Figure 4). The step angle is estimated to be approximately 37° on the left side and 40° on the right side. The distance from the first Mach stem to the original point is 1.80 km. Similarly, the step angle calculated based on the second and third Mach stem is approximately 20° on the left and 23° on the right. The travel distance is 1.73 km.

Most internal waves observed in the SW’06 experiment have amplitudes between 4 m and 10 m and wave speeds between 0.62 m/s and 0.80 m/s [20]. For satellite-observed internal waves, we can determine their phase speeds accurately if in-situ measurements are nearby or two successive internal wave groups from the same origin are observed in a single image. Unfortunately, internal waves in the observed interaction patterns satisfy neither of these two conditions. However, since the satellite images were collected during the SW’06 experiment, it is highly likely that the satellite observed internal waves lie in the same phase speed range. As a result, the time of developing the first Mach stem is estimated between 37 min and 48 min. Similarly, the time of developing the second one is between 36 min and 46 min. In our study area, the upper layer depth is about 15 m calculated from the conductivity-temperature-pressure data.
obtained during the experiment. As a result, the wave amplitude normalized by the upper layer depth in a two-layer stratified water is between 0.27 and 0.67. According to Miles’ theory, the post-interaction angle should be between 102° and 162°, which is much larger than the observed 78°. The post-interaction normalized amplitude is 0.05. The step angle is between 10° and 20°. Obviously, the internal wave amplitudes in the MAB do not satisfy the small amplitude requirement for Miles’ theory. Tanaka [7] studied Mach interaction at a normalized wave amplitude of 0.3, which lies within our amplitude range in the MAB. The post-interaction angle and normalized amplitude in Tanaka’s simulation is found to be 85° and 0.09, respectively. The step angle is 6°. By looking at the image brightness, we can qualitatively say that the post-interaction amplitude is much smaller and the Mach stem amplitude is larger than the pre-interaction wave amplitude. Our observation agrees more with Tanaka’s numerical results except for the step angle. Since internal wave amplitudes usually rank in order, the length of the first Mach stem at the location of the second Mach stem may have been longer than we measure from the image, which would result in the step angle being larger than our estimate.

Based on Tanaka’s numerical results and Wang and Pawlowicz [8]’s finding that the likelihood of Mach interaction is overestimated by Miles’ theory, we think that the large amplitudes in the MAB tend to prevent the Mach interaction from occurring. Therefore, if the post-interaction angle and amplitudes are very close to the pre-interaction ones in an observed interaction pattern in our study area, we treat it as a regular interaction.
4.2 Regular interactions

During the SW’06 experiment, another clear interaction pattern is observed in the SPOT-2 HRV-PAN image taken on August 17, 2006 (Figure 5). The brightness signatures of the internal waves decrease in the interaction zone, which indicates that the wave amplitude also decreases. Figure 6(a) shows a sketch of the interaction pattern of the first three waves in both packets. Packet C stands for the wave packet on the left and packet D stands for the wave packet on the right. The naming rules for the pre- and post-interaction as well as wave crests in the interaction zone are the same as in Figure 3. The interaction process is illustrated in Figure 6(b) using the first waves in both packets. The pre-interaction angle, between C1 and D1, is approximately 37° and the post-interaction angle, between c1 and d1, is approximately 42°. The two angles are very close to each other so the waves C1 and D1 have a regular interaction. This interaction pattern is a regular interaction with an amplitude decrease in the interaction zone. The post-interaction wave c1 continues to interact with the second wave D2 in the right packet. The resulting post-interaction wave cc1 then interacts with the third wave D3, which results in the post-interaction wave ccc1. Similarly, the post-interaction wave d1 continues to interact with the second wave C2 in the left group, which results in the post-interaction wave dd1. The hexagonal pattern resulting from the interaction is shown in Figure 6(c), which agrees with the pattern found from periodic solutions to the KP equation [21].

In Figure 7, another interaction pattern is observed in the RADARSAT-1 image taken on August 01, 2006. The image intensity increases in the interaction zone, which indicates that the wave amplitude also increases. The interaction pattern looks similar to Mach interaction, having
a merged front in the interaction zone. A sketch of the interaction of the first waves is shown in Figure 8. The pre-interaction angle, between E1 and F1, is approximately 49° and the post-interaction angle, between e1 and f1, is approximately 47°. This interaction pattern is a regular interaction with an amplitude increase in the interaction zone.

Different amplitudes of the incoming internal waves and different angles between their propagation directions will result in different interaction patterns. In the following, the two-soliton solution (Equations (9)-(11)) is applied to simulate the two interaction patterns in Figures 5 and 7. In order to simulate the internal wave interaction pattern by the KP model, we need to know the amplitudes and the interaction angles of the incoming waves. The angles for the two cases are approximately 37° and 49° measured from the images; however, only relative amplitudes of the two incoming waves are used because no in-situ measurements are available near the observed interaction patterns. As a result, this study mainly focuses on the qualitative aspects of the interaction characteristics. All parameters needed to simulate the two observed interaction patterns are listed in Table 1.

The interaction pattern in Figure 5 is very clear and strong. This is partly due to the 10 m resolution of the SPOT-2 image. Figure 9 shows the 3-D and 2-D simulations of the two-soliton interaction pattern in Figure 5. For a clear view, $-u$ is used instead of $u$. We can see that the amplitude in the interaction zone decreases significantly when $A_{12} < 1$, a positive phase shift, which fits well with the pattern we observed in Figure 5. The resulting relative amplitude in the interaction zone is only 23% compared with the smaller incoming wave’s amplitude. This pattern looks similar to case C discussed by Wang and Pawlowicz [8]. The case C pattern lies in
Miles’ Mach interaction category like the other cases in their study, but does not show any merged front in the interaction zone. Wang and Pawlowicz [8] give two possible explanations. The first possibility is that it is a reflection problem because the wave parameters such as post-interaction amplitudes and angle are not dramatically different from those cases with merged fronts. A second possibility is that the wave amplitude or the difference in wave amplitudes may be large enough to be above Miles’ criteria of phase conservation and the case is fundamentally an oblique overtaking. Based on the KP model, we can see that this pattern is actually due to a regular interaction with an amplitude decrease in the interaction zone.

The internal wave signatures in the SAR image of Figure 7 are not as clear as in the optical images, but the leading solitons still show a clear interaction pattern. In Figure 10, we can see that the amplitude increases significantly in the interaction zone and the interaction has a negative phase shift ($A_{12} > 1$), which fits well with the pattern we observed in Figure 7. The resulting amplitude in the interaction zone is 3.56 times that of the incoming wave. The two simulations shed some light on future studies of regular internal wave interaction patterns in satellite images. If we observe clear two-soliton interaction patterns in a satellite image, even if we do not know the amplitudes of the incoming solitons, we can deduce how the amplitudes change in the interaction zone, just based on the patterns themselves.

4.3 Interactions without phase shifts

Besides all the interaction patterns with clear phase shifts we discussed above, we also observed several interaction patterns without phase shifts. One example is found in a RADARSAT-1 image from August 13, 2006 (Figure 11). The interaction angle is approximately $63^\circ$. A possible
reason for such pattern may be very small phase shifts at certain wave amplitudes and interaction
angles, which results in the narrow width of the hump in the interaction zone that our image
cannot resolve. The simulation by the KP model is illustrated in Figure 12. We can see that the
hump in the interaction zone has a very narrow width in the 3-D interaction pattern in Figure
12(a) and we can barely see any phase shifts in the 2-D interaction pattern in Figure 12(b).
Another possible reason is that the observed interaction pattern is still in the development stage
and has not reached the stable stage with clear phase shifts described by the two-soliton solution
for the KP equation.

5. Conclusions

High resolution satellite images provide a useful and efficient means to study internal wave
interaction patterns. Some interaction characteristics can be directly observed, such as
pre-interaction and post-interaction angles, phase shifts, and qualitative amplitude changes in the
interaction zone. In this paper, we have observed and studied four types of internal wave
interactions in the MAB. We can determine the interaction type by just examining the patterns in
the images. If the post-interaction angle is much larger than the pre-interaction angle, we have a
case of Mach interaction. The length of the stem increases linearly with the propagation distance.
The Mach stem amplitude is larger and the post-interaction wave amplitudes are much smaller
than the incoming internal wave amplitudes. If the post-interaction angle is very close to the
pre-interaction angle, we have a case of regular interaction. There are two different types. One
type results in a merged front in the interaction zone and looks similar to Mach interaction. The
amplitude in the interaction zone is always larger than the incoming wave amplitudes. However,
the post-interaction amplitudes are close to the incoming wave amplitudes and the length of the merged front is constant and does not increase with propagation distance. The other type does not result in a merged front in the interaction zone. The amplitude in the interaction zone is much smaller than the incoming wave amplitudes and the post-interaction amplitudes are close to the incoming wave amplitudes. All three types above show clear phase shifts due to the interaction. We also observe interaction patterns without prominent phase shifts. A possible explanation is that the phase shifts are very close to zero at certain incoming wave amplitudes and interaction angles. In other words, the width of the hump in the interaction zone is too small to resolve. Another possible reason is that the observed interaction pattern is still in the development stage and has not reached the stable stage with clear phase shifts.

Much attention goes to the interactions that result in larger amplitudes in the interaction zone. In Miles’ theory, the maximum wave amplitude is 4 times that of the incoming wave. The merged front resulting from internal wave interaction can have a length of over 2 km, as shown in Figure 2, and it can carry large mass, momentum, and energy over a long distance. Consequently, internal waves with such large amplitudes can be hazardous to underwater structures. This paper presents a qualitative study of these interactions patterns. In the future, either in-situ measurements of amplitudes of pre-interaction waves, post-interaction waves, and the waves in the interaction zone are needed to compare with the results of the KP model, or a method to estimate internal wave amplitudes from their signatures in satellite images needs to be developed. Xue et al. [22] developed a method based on the higher order KdV equation to estimate large amplitude internal solitary waves over 10 m in the MAB, but a method to estimate
wave amplitude in the range of 4 m to 10 m is still needed. Moreover, numerical studies of internal wave interactions are needed to compare the interaction patterns observed in the satellite image and get a better understanding of the interaction processes.
References


Biography

Jingshuang Xue (M’12) received the B.S. degree in physical geography from Nanjing University, Nanjing, China, in 2007 and the M.S. degree at the University of Miami’s Rosenstiel School of Marine and Atmospheric Science, Miami, FL, 2010.

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Hans C. Graber received his B.E. in Civil Engineering (magna cum laude) with a minor in Mathematics from the City College of New York (CCNY) in 1977 and his M.S. and Sc.D. degrees in Coastal Engineering / Hydrodynamics from the Massachusetts Institute of Technology (MIT) in 1979 and 1984, respectively.

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His research interests focus on experimental, theoretical and numerical studies of ocean surface wave dynamics, radar remote sensing, maritime domain awareness, air-sea fluxes including boundary-layer dynamics and hurricane and storm surge predictions and has been involved in many national and international projects and experiments on these topics.

Roland Romeiser (M’00) received the Dipl.-Phys. degree from the University of Bremen, Bremen, Germany, in 1990 and the Dr.rer.nat. and Habilitation degrees from the University of Hamburg, Hamburg, Germany, in 1993 and 2007, respectively. He was a Project Scientist with the Institute of Oceanography, University of Hamburg, from 1990 to 1998 and a permanent Staff Scientist with the same institute from 1999 to 2009. From August 1998 to July 1999, he spent a year with the Applied Physics Laboratory, Johns Hopkins University, Laurel, MD, as a Feodor Lynen Fellow of the Alexander von Humboldt Foundation. Since 2008, he has been an Associate Professor with the Rosenstiel School of Marine and Atmospheric Science of the University of Miami, Miami, FL. He has wide experience in the remote sensing of ocean currents,
waves, and winds by various microwave sensors. He has been involved in many international remote sensing projects and experiments. His current research focuses on the theoretical modeling of SAR and InSAR signatures of spatially varying ocean surface currents and the development and evaluation of current retrieval techniques.

Dr. Romeiser has organized and chaired several sessions at international remote sensing conferences. Since fall 2000 he is an associate editor of the IEEE Journal of Oceanic Engineering.

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He worked in the Software Department at OceanWaveS GmbH, Lüneburg, Germany, where he was involved in several national and international research projects on nautical X-band radar applications (2002-2007). Before joining the University of Miami Ph.D. program, he had a position as Visiting Researcher at the Rosenstiel School's division of Applied Marine Physics, where he worked on the development of algorithms to extract wind, internal wave, surface current, and individual wave information from shipborne nautical radar data (2007-2009). His current research focuses on nautical X-band radar remote sensing and air-sea interaction processes.
Figure 1. Locations of four observed interaction patterns over the bathymetry of the MAB. The rectangles (a), (b), (c), and (d) outline the locations of the cases shown in Figures 2, 5, 7, and 11.
Figure 2. An interlocked, zipper-like internal wave interaction pattern observed in the SPOT-4 HRV-PAN image (17.6 km × 12.1 km) taken on August 05, 2006 (the rectangle (a) in Figure 1).
Figure 3. (a) Sketch of the interaction pattern of the first three waves in Figure 2. (b) Sketch of the interaction of the first waves in both packets ($\psi=45^\circ$, $\psi_p=78^\circ$), the extended dashed lines of A1 and B1 are the approximate wave crest locations without interaction.
Figure 4. The lines connect the edges of the merged fronts on both sides to get the generation point for the first and second merged fronts (a) and the second and third merged fronts (b).
Figure 5. An internal wave interaction pattern observed in the SPOT-2 HRV-PAN image taken on August 17, 2006 (15.5 km × 10.0 km) in the MAB (the rectangle (b) in Figure 1).
Figure 6. (a) Sketch of the interaction pattern of the first three waves in Figure 5. (b) Sketch of the interaction of the first waves in both packets ($\psi=37^\circ$, $\psi_p=42^\circ$). (c) The hexagonal pattern resulting from the wave interactions.
Figure 7. An internal wave interaction pattern observed in the RADARSAT-1 image taken on August 01, 2006 (18.8 km × 15.3 km) in the MAB (the rectangle (c) in Figure 1).
Figure 8. Sketch of the interaction of the first waves in Figure 7 ($\psi=47^\circ$, $\psi_p=49^\circ$), the extended dashed lines of E1 and F1 are the approximate wave crest locations without interaction.
Figure 9. (a) Snapshot of the two-soliton solution for the KP equation at $t=0$ with the parameters for Figure 5 from Table 1. (b) The corresponding 2-D phase shift pattern.
Figure 10. (a) Snapshot of the two-soliton solution for the KP equation at $t=0$ with the parameters for Figure 7 from Table 1. (b) The corresponding 2-D phase shift pattern.
Figure 11. An internal wave interaction pattern without phase shifts observed on the RADARSAT-1 image taken on August 13, 2006 (20.5 km × 16.6 km) in the MAB (the rectangle (d) in Figure 1).
Figure 12. (a) Snapshot of the two-soliton solution for the KP equation at $t=0$ with the parameters for Figure 11 from Table 1. (b) The corresponding 2-D phase shift pattern.
Table 1. Parameters for the simulation of the patterns in Figures 5, 7, and 11.

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