Current variability on a narrow shelf with large ambient vorticity

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[1] Surface and subsurface currents and stratification were observed on and near the narrow shelf off Fort Lauderdale, Florida, in June–August 1999. The Florida Current flowed past, occasionally on the shelf, with speeds at times exceeding 2 m s⁻¹ only 8 km offshore. The typical vorticity associated with the lateral shear of the Florida Current was 4f, where f is the local Coriolis parameter. Two dominant modes of higher frequency current variability were embedded in this low-frequency flow, a 10-hour signal with amplitudes of up to 0.5 m s⁻¹ and an equally as strong, almost rectilinear signal with a 27-hour period, the same as the local inertial period. Both signals were propagating northward along the shore with wavelengths of ~27 and ~170 km for the 10- and 27-hour signals, respectively, and with corresponding phase speeds of 0.85 and 1.7 m s⁻¹. The phase trend in east-west direction of the 10-hour signal is consistent with unstable, growing waves. The 10- and 27-hour signals appear dominantly barotropic near the coast out to depths of at least 50 m, but the 10-hour signal displayed a 180° phase change between surface and bottom farther away from the shelf break at 160 m depth. The two dominant signals cannot be attributed to barotropic instability.

INDEX TERMS: 4219

1. Introduction

[2] Energetic, highly nonlinear and complex flow regimes arise along the western edge of western boundary currents such as the Florida Current and the Gulf Stream. These currents follow the steep bottom terrain along the shelf break, which separates the deep ocean from the coastal flow regimes. The boundary currents tend to intrude onto the shelf on 3- to 7-day periods. In the littoral ocean, cooler, less saline water interacts with the warm, more saline subtropical water on spatial scales of kilometers, forming filaments, rips, eddies and other mesoscale variability in the oceanic flows [Marmorino and Trump, 1994]. Fluctuations of the western boundary currents force, trap, and advect a spectrum of transient features such as submesoscale vortices and internal waves that have an impact on mixing between differing water masses [Meid et al., 1996; Shay et al., 1998a].

[3] Of particular relevance for this paper is the width of the shelf, which is well known to have an impact on the littoral circulation. In the Southern Atlantic Bight, for example, M₂ tidal amplitudes increase with the northward increase of the shelf width from Florida into Georgia, as noted by Redfield [1958]. In that study the shelf is wide, with the shelf break and the western boundary current being located well offshore. However, little appears to be known about the impact of strong adjacent currents on a narrow shelf, the topic of this paper. The continental shelf in the vicinity of Fort Lauderdale, Florida, is narrow and shallow, the shelf break being marked by the “Third Reef,” a coral reef only 2.5 km offshore. Depths inside of the reef do not exceed 22 m (Figure 1), and the coastline is quite straight, stretching approximately in the north-south direction (Figure 2a). We analyze here how the circulation on and near the shelf is affected by the presence of the Florida Current (FC) with principally northward velocities exceeding 2 m s⁻¹ at times (Figure 3b).

[4] Measuring ocean surface currents with Doppler radar systems, such as the Ocean Surface Current Radar (OSCR), has opened a wide range of new possibilities for coastal oceanography [Prandle, 1987; Shay et al., 1995]. This technique is based on the backscattering of radio waves by resonant surface waves of one half the incident radar wavelength. This Bragg scattering effect [Stewart and Joy, 1974] results in two discrete peaks in the Doppler spectrum. In the absence of a surface current the spectral peaks are symmetric, and their frequency (σ) is offset from the origin by an amount proportional to 2c₀λ⁻¹, where c₀ represents the phase speed of the surface wave and λ is the radar wavelength. Surface currents displace the Bragg peaks in the Doppler spectrum by Δσ = 2Vᵣλ⁻¹, where Vᵣ is the radial current component along the radar beams. The two-dimensional velocity vector is resolved by using two radar stations.

[5] Herein we analyze the first extensive measurements using the OSCR system in its high-resolution, very high
frequency (VHF) mode during the July/August 1999 "Four-Dimensional Current Experiment" (4D Experiment). Our data reveal substantial coastal ocean current variability, even during quiescent atmospheric conditions. Surface flows at 250-m resolution in a $7 \times 8$ km domain reveal vortices, frontal lobe-like structures, and Florida Current intrusions over the shelf break at 1- to 3-day intervals. Shay et al. [2000] detail the propagation of a submesoscale vortex with closed streamlines of 2 km diameter and currents of $0.3–0.5$ m s$^{-1}$ at 1.25–1.5 km from its center. Such coherent scales have never been observed at such high resolution with surface current radar measurements. The high quality of the summer 1999 OSCR observations has been assessed by Shay et al. [2002].

[6] We combine surface current observations from VHF radar with shipboard and moored current and stratification measurements in examining currents on the shelf and over the shelf break, focusing on two dominant, narrow-band signals of 10- and 27-hour periods, respectively. The strength of the 10- and 27-hour signals as described in section 5.2 and 5.3, respectively, makes them relevant in practical terms of navigation and sediment transport. An analysis of the weak tidal flows as well as investigations of other frequency bands are deferred to future studies. The observations discussed herein are outlined in section 2, which is followed by some necessary background information on stratification and low-frequency currents. In section 4 we analyze the space-time structure of surface currents in some generality before concentrating on the 10- and 27-hour signals in section 5. The paper concludes with a summary and a discussion of the possible nature of these signals.

2. Observations

[7] In this paper we use a subset of the observations acquired as parts of the 4D Experiment in the newly established South Florida Ocean Measurement Center off Dania Beach near Fort Lauderdale from July to August 1999 (Table 1). We focus on the surface currents and only mention the stratification to the degree necessary to assess its effect on the circulation. The record from the
Navy 150-m mooring has large gaps in time and is therefore used only sparingly.

2.1. OSCR Measurements

The OSCR radar system was deployed from 25 June to 10 August 1999, yielding a 29-day continuous time series of vector surface currents from 9 July to 7 August 1999 at 20-min intervals. The system consisted of two VHF radar transmitter/receiver stations operating at 49.9 MHz and sensing electromagnetic signals scattered from surface gravity waves with wavelengths of 3 m. The VHF radar system mapped coastal ocean currents over a 7 \times 8 km domain with a horizontal resolution of 250 m at 700 grid points, shown in Figure 2a. The northern, master, radar site was located at Port Everglades Inlet, while the southern, slave, site was located at Hollywood Beach, the baseline length being ~6 km. Each site consisted of a four-element transmit array and a thirty-element receiving array spread over 85 m sensing electromagnetic signals scattered from surface gravity waves with wavelengths of 3 m. The VHF radar system mapped coastal ocean currents over a 7 \times 8 km domain with a horizontal resolution of 250 m at 700 grid points, shown in Figure 2a. The northern, master, radar site was located at Port Everglades Inlet, while the southern, slave, site was located at Hollywood Beach, the baseline length being ~6 km. Each site consisted of a four-element transmit array and a thirty-element receiving array spread over 85 m

<table>
<thead>
<tr>
<th>System</th>
<th>Instrument</th>
<th>Domain, Grid, Location</th>
<th>Depth, m</th>
<th>Deployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSCR</td>
<td>surface Doppler radar</td>
<td>see Figure 2a, $\Delta t = 20$ min, $\Delta x = 250$ m</td>
<td>see Table 2</td>
<td>9 July 1999 to 7 August 1999</td>
</tr>
<tr>
<td>R/V Stephan</td>
<td>ADCP, CTD</td>
<td>see Figure 2b, $\Delta t = 1$ m</td>
<td>since 1994</td>
<td>see Table 2</td>
</tr>
<tr>
<td>Navy 150-m mooring</td>
<td>ADCP</td>
<td>$\lambda = -80.0942^\circ, \phi = 26.0695^\circ$, $\Delta t = 15$ min, $\Delta x = 1$ m</td>
<td>158.5</td>
<td>since 1994 with large gaps</td>
</tr>
<tr>
<td>NE mooring</td>
<td>ADCP, CTDs,</td>
<td>$\lambda = -80.0768^\circ, \phi = 26.0695^\circ$, $\Delta t = 15$ min, $\Delta x = 1$ m</td>
<td>50</td>
<td>13 July 1999 to 12 August 1999</td>
</tr>
<tr>
<td>SW mooring</td>
<td>ADCP, CTDs,</td>
<td>$\lambda = -80.0918^\circ, \phi = 26.0327^\circ$, $\Delta t = 15$ min, $\Delta x = 1$ m</td>
<td>20</td>
<td>11 August 1999</td>
</tr>
</tbody>
</table>

Table 1 covers only a subset of all measurements acquired during the 4D Experiment. NE and SW moorings were deployed by personnel from the University of South Florida. The South Florida Test Facility of the U.S. Navy maintains the Navy 150-m mooring.
length. Array orientations were $37^\circ$ true for the master and $160^\circ$ for the slave [Shay et al., 2002].

[9] The manufacturer states accuracies of the radial and vector speeds of 0.02 and 0.044 m s$^{-1}$, respectively, the directional resolution being $5^\circ$. The accuracy of OSCR measurements has been examined by Shay et al. [1995], Chapman et al. [1997], and Shay et al. [1998b]. Their comparisons of OSCR-derived surface currents with moored and shipboard sensors show RMS differences of 0.07 m s$^{-1}$ over a velocity range of 1 m s$^{-1}$.

### 2.2. Other Measurements

[10] Subsurface currents and stratification were measured from the R/V Stephan during seven short cruises listed in Table 2. The acoustic Doppler current profiler (ADCP) was a RD-Instruments 600-kHz, 5-beam, broadband system mounted over the side of the R/V Stephan. We used bin widths of 1 m (0.5 m in 4D02 and 4D07) and averaged the data along the ship track over distances of $\sim$150 m in order to suppress instrument noise (see Figure 2b). All shipboard ADCP current data rely on bottom tracking, which extended to water depths of 80–90 m. Velocity data reached to depths of 40–50 m. The internal compass of the ADCP was calibrated using GPS navigation data. The compass worked well because the R/V Stephan has an aluminum hull.

[11] While OSCR provides $u$ and $v$ in the $(x, y, t)$ domain at the surface, moored ADCPs cover the $(z, t)$ domain at specific points in the horizontal. The moored current observations utilized herein are summarized in Table 1. Information on the stratification of the water column is provided by a total of 151 conductivity-temperature-depth probe (CTD) casts acquired mostly at the corners of the boxes shown in Figure 2b. We used an Ocean Sensors OS500 CTD during cruises 4D02 to 4D05 and a SeaBird SeaCat CTD thereafter. The pump of the SeaCat failed during 4D06 such that CTD data from part of 4D06 and from 4D07 have poor vertical resolution.

### 2.3. Notation

[12] Standard Cartesian coordinates are given by $(x, y)$, corresponding to east and north. As usual, the corresponding velocity components are $(u, v)$. We denote longitude and latitude by $\lambda$ and $\phi$, respectively, and use $f$ as the Coriolis parameter for a latitude of $\phi = 26^\circ$N. The water depth is $H$. In a bit of simplification, we take the longitude of the beach at the surface, moored ADCPs cover the $(z, t)$ domain at specific points in the horizontal. The moored current observations utilized herein are summarized in Table 1. Information on the stratification of the water column is provided by a total of 151 conductivity-temperature-depth probe (CTD) casts acquired mostly at the corners of the boxes shown in Figure 2b. We used an Ocean Sensors OS500 CTD during cruises 4D02 to 4D05 and a SeaBird SeaCat CTD thereafter. The pump of the SeaCat failed during 4D06 such that CTD data from part of 4D06 and from 4D07 have poor vertical resolution.

### 3. Background Conditions

[13] Even casual considerations of flow dynamics indicate that the current fluctuations analyzed herein were likely affected by the advection and the relative vorticity associated with the FC as well as by the stratification of the water column. We begin our discussion with the latter.

#### 3.1. Stratification and Baroclinicity

[14] The water column in the study area was strongly stratified with significant contributions from both temperature ($T$) and salinity ($S$). Figure 4 shows examples of CTD casts and velocity profiles taken on the shallow shelf and just off the shelf break. Characteristic, though variable, features include step-like potential density profiles ($\sigma$) and strong stratification right to the surface. “Step-like” means that profiles consist of a series of alternating, clearly identifiable and coherent low-gradient and high-gradient layers. The steepness in Figure 4d has been reduced by smoothing in the vertical. Quantitatively, the stratification is characterized by an overall depth-time average squared buoyancy frequency of $N^2 = 1.7 \times 10^{-3}$ s$^{-2}$, ($N = 1$ cph), with bootstrap confidence limits of $[5.8 \times 10^{-4}$, $3.1 \times 10^{-3}]$ s$^{-2}$. For details of the bootstrap technique, the reader is referred to Efron and Gong [1983]. For general interest, Figure 4 also shows the gradient Richardson number $Ri$.

[15] Hence the water column tended to be strongly stratified, and, correspondingly, the velocity field tended to be sheared in the vertical and horizontal as illustrated in Figures 3 and 4. Much of the vertical shear was associated with the geostrophically balanced FC and resided at low frequencies in the spectral domain. At higher frequencies, barotropic current variations dominated over baroclinic variations on the shallow shelf and the upper continental slope. Among these (more or less) barotropic variations are the 10- and 27-hour signals, the focal point of this paper. Table 3 lists the variances of the $u$ and $v$ current components on the shelf at depths of $<20$ m (“E”), on the upper continental slope at depths below 80 m (“W”), and at the “Navy 150-m” mooring at 159 m depth. The variances are those of the depth-average flow and of the residual of the current variation after removing its depth average. As shown in Table 3, the depth-average current variance exceeded the residual by at least an order of magnitude on the shelf and by factors of 2–4 at 159 m depth.

![Figure 4](image-url)
Table 3. Variances of East ($u$) and North ($v$) Current Components From North-South Shipboard ADCP Sections

<table>
<thead>
<tr>
<th></th>
<th>Depth Average, m$^2$ s$^{-2}$</th>
<th>Residual, m$^2$ s$^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^2$ (E)</td>
<td>$1.5 \times 10^{-2}$</td>
<td>$1.6 \times 10^{-3}$</td>
</tr>
<tr>
<td>$v^2$ (E)</td>
<td>$1.6 \times 10^{-4}$</td>
<td>$1.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>$u^2$ (W)</td>
<td>$3.2 \times 10^{-2}$</td>
<td>$4.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$v^2$ (W)</td>
<td>$4.7 \times 10^{-2}$</td>
<td>$1.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$u^2$ (Navy)</td>
<td>$1.1 \times 10^{-2}$</td>
<td>$5.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$v^2$ (Navy)</td>
<td>$2.5 \times 10^{-2}$</td>
<td>$6.5 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

*Sections are on the shoreward side (W) of the boxes shown in Figure 2b and on the offshore side (E) and from the Navy 150-m mooring (Navy).

Even though the principal observational basis of our analysis is surface current observations, we are able to discuss barotropic processes owing to the described dominance of barotropic over baroclinic currents. Figure 5 further substantiates this statement by demonstrating a high correlation between surface and depth-averaged currents, $u_{surf}$ and $u_{davg}$, and $v_{surf}$ and $v_{davg}$, respectively. The largest excess of $v_{surf}$ over $v_{davg}$ occurred at large $v_{surf}$, a feature associated with the slowly varying FC and located geographically offshore of the shelf. The profiles shown in Figures 4d and 4e are in this category and are contained in Figure 5.

It is important for the interpretation of the observed current signals that the dominance of barotropic over baroclinic current fluctuations occurred in the relatively shallow waters inside and just outside of the shelf break. This does not preclude that these signals could be the upper ocean expression of comparatively deep-reaching baroclinic features. Section 4.2 notes expressions of baroclinicity in data from the Navy 150-m mooring at the 160-m-depth contour.

### 3.2. Low-Frequency Currents and Ambient Vorticity

The FC flowed fairly close to the shelf break, at times reaching velocities of $>2$ m s$^{-1}$ within 8 km of the coast (Figure 3b). The associated cross-shelf shear is equivalent to an ambient largescale vorticity of a time-averaged magnitude of up to 4$\bar{f}$ with peaks reaching 12$\bar{f}$ (Figure 6). Note that the ambient vorticity implies cyclonic rotation while the Coriolis force induces anticyclonic rotation. The local inertial period being 27 hours, a vorticity of 4$\bar{f}$ linearly affects motions with periods of just a few hours, and, consequently, the 10-hour signal examined here has to be greatly affected also. Because the contribution of $\partial u/\partial y$ to the low-passed relative vorticity was small except near the coast during some periods, we use $\partial v/\partial x$ as a proxy for the largescale relative vorticity. The low-pass filter used to generate the data shown in Figure 6 has a cut-off period of 80 hours.

Figure 6 shows how the velocity and ambient vorticity of the FC was modulated at low frequencies. The sign reversal of $f^{-1}$ $\partial v/\partial x$ near year day 215 and the extremely large positive $f^{-1}$ $\partial v/\partial x$ at year days 209–213 stand out. A comparison with Figure 3 shows that the latter event was associated with the largest northward flow observed and that the year day 215 event was associated with southward flow across the domain. Within the small spatial aperture of the OSCR domain, we are unable to distinguish between onshore-offshore meandering and pulsation of the FC that has been observed elsewhere along the south Florida coast along with mesoscale eddy activity [Johns and Schott, 1987; Zantopp et al., 1987; Lee et al., 1995]. We compute $\partial v/\partial x$ from objectively analyzed surface current fields $\mathbf{u}(x, y)$ as explained in section 4.

As a basis for the discussions of barotropic flow stability found in section 6 and Appendix A, we show the component of the potential vorticity associated with the mean shear, $\mathbf{Q} = (\partial \mathbf{V}/\partial x)$, in Figure 6f and depict the water depth in Figure 6e. The mean potential vorticity peaked at the shelf break near ~80.08$^\circ$ longitude in response to a maximum of $\partial \mathbf{V}/\partial x$ just offshore of the shelf break. The structure of $\mathbf{Q}(x)$ depends critically on the FC intruding onto the shelf to a degree. Figure 6f shows that the presence, absence, and degree of such intrusions were highly variable in time.

We use surface currents instead of depth-averaged currents in estimating potential vorticity, and thus $\mathbf{Q}$ is biased. The magnitude of this bias can be estimated from taking the ratio of $v_{surf}$ and $v_{davg}$ in Figure 5a. This ratio reaches a factor of 1.5 at some large $v$. The positive bias in $\mathbf{Q}$ increases with increasing $x$ because the effect of baroclinicity becomes more pronounced with increasing water depth. Hence Figure 6f needs to be viewed in a qualitative sense. Appendix A notes why we do not include the Coriolis parameter $f$ in the definition of $\mathbf{Q}$.

### 4. Space-Time Structure of Surface Currents

#### 4.1. Time and Space Domain

The time series shown in Figure 3 visualize typical temporal variations of surface current. In order to also provide the reader with a visual impression of spatial variability patterns of surface currents, we show one 10-hour time period, year day 208, 0000–1000 UTC, in Figures 7a–7f. This period has been selected because of strong activity at the 10-hour period, a feature exploited in section 5.2. For the sake of clarity of patterns, Figure 7 is based on objectively analyzed surface current fields introduced further below. To characterize the spatial current variability, we note that there was pronounced variability over the 10-hour time span and that the flow showed substantial vorticity and divergence changing on a timescale of the order of an hour and on spatial scales of the order of a kilometer.

![Figure 5. Near-surface (3.7 m) currents versus depth-averaged currents from the shipboard ADCP for (a) $v$ component and (b) $u$ component.](image)
We begin a quantitative analysis of the spatial and temporal variability of surface currents in terms of their autocorrelation properties. Three-dimensional (3-D) correlation functions for $u$ and $v$ were calculated from the edited OSCR surface velocity data. An example of this function, at zero temporal lag, is shown in Figure 8. The autocovariance function, with velocity variance $s^2$, 

$$R(t) = \sigma^2(1 - \epsilon_n^2) \cos(2\pi t/P)e^{-t/\tau_e}$$

is fit to each of the 3-D correlation functions using the temporal lags from 0, 2, 4, ..., 24 hours at zero spatial lag. Note that zero spatial lag includes all spatial lags from between 1 and 1 km. The normalized variance $\epsilon_n^2$ is the variance of subgrid-scale processes divided by the velocity variance not contained in the largest spatial scales that were removed by bicubic spline fits to the data. The period $P$ equals four times the zero-crossing scale of the correlation function, and $\tau_e$ is the e-folding scale or the turbulent timescale. This covariance function, based on a second-order autoregressive process, contains a wave component and a turbulent component and produces good fits to the observed covariance functions.

The three parameters ($\epsilon_n^2$, $P$, and $\tau_e$) are determined using the feature-based technique described by Mariano and Chin [1996], which finds the value of $R(t)$ at zero lag $= \sigma^2(1 - \epsilon_n^2)$. It further finds the zero-crossing scale of $R (= P/4)$ by determining where $R(t)$ changes from positive to negative and finds the e-folding scale $\tau_e$ from the first two parameters, an initial guess consisting of the lag at which $R(t)$ is 1/e of its initial value, with the best fit being determined in a least-squares sense. Extreme outlier values sometimes appear when the autocorrelation function is noisy or does not decay with large temporal lag. These values are removed from the analysis.

Timescales are first determined by evaluating the integral of $R(t)$,

$$\int_0^\infty R(t)dt = \sigma^2(1 - \epsilon_n^2) \frac{\tau_e}{2} \sqrt{\pi} \exp\left(-\frac{\pi \tau_e^2}{\sigma^2}\right),$$

using the estimated three parameters and $\sigma^2$. Then the integral timescale is

$$T = (1 - \epsilon_n^2) \frac{\tau_e}{2} \sqrt{\pi} \exp\left(-\frac{\pi \tau_e^2}{\sigma^2}\right).$$

Figure 6. Low-passed along-shelf (north) current component, its cross-shelf shear, and potential vorticity at $\phi = 26.06^\circ$. (a) Time-longitude contours of $v$ and (b) its time average $V$. (c) Water depth. (d) Time-longitude contours of $f^{-1} \partial V/\partial x$ and (e) its time average. (f) Rough estimate of the component of the mean potential vorticity related to $\partial V/\partial x$. Contour interval in Figure 6d is 1. Heavy lines denote zero contours in Figures 6a and 6d.
Spatial maps of the normalized subgrid-scale and measurement variance for $u$ and $v$ are shown in Figures 9a and 9b. These maps are an estimate of normalized variance on space scales and timescales on the order of 1 km and 1 hour, respectively. Subgrid-scale and measurement variability contributed from 25 to 30% of the residual cross-shelf velocity variance and from 10 to 25% for the residual alongshelf component.

Maps of the dominant period are shown in Figures 9c and 9d. The 10-hour period, which also emerges by other methods and from other data, is clearly seen over most of the domain. However, there is a significant trend of the $v$ velocity period increasing from 10 hours nearshore to over 3 days offshore, presumably due to the lower frequency meandering of the Florida current.

The maps of e-folding, or turbulent, timescales for $u$ and $v$ reveal that on the average, space scales and timescales of $v$ are larger than those of $u$ and that both $u$ and $v$ timescales increase offshore. The temporal e-folding scales increase from 1 to 2 hours for $u$ and from 2 to 8 hours for $v$. The Eulerian integral timescale ($T_e$) is 1.5–2 hours for $u$, and it increases from 2 to 10 hours offshore for $v$ (not shown). Thus hourly sampling is required, at a minimum, to prevent aliasing in estimates of surface velocity in this coastal regime. Accurate derivative estimates require about three times the sampling rate or $\approx 40$ min, consistent with the OSCR sampling interval.

4.2. Frequency Domain

Power spectra of the surface currents shown in Figure 3 are presented in Figure 10. The spectra reveal three dominant frequency bands, low frequency, “27 hours,” and “10 hours”. A low-frequency signal with $\omega \approx 0.015$ cph was associated with the fluctuating FC. This signal is prominent only in $v$, as is a narrow-band signal with periods near the local inertial period of 27 hours ($\omega \approx 0.037$ cph). We dub the third energetic frequency band at 0.08 cph $\leq \omega \leq 0.1$ cph the “10-hour signal,” noting that it includes the semidiurnal tides.

A careful look at Figure 10 also reveals significant energy at frequencies $\omega > 10$ cph. These high-frequency signals, as well as the tides, will be analyzed elsewhere. With respect to the latter, it is herein sufficient to state that tidal analysis shows semidiurnal tidal currents to be much smaller than the 10- and 27-hour signals. Figure 10 further shows that surface currents had less energy near the coast, especially in the low-frequency component, and that the 10- and 27-hour signals had maximum variance near the middle of the OSCR domain. Commensurate with topographic effects, the along-shelf flow ($v$) generally had more energy than the cross-shelf flow. We can also show that the anticlockwise, cyclonic, rotational current component dominated over the clockwise, anticyclonic, component at practically all frequencies, especially at the 27- and 10-hour periods. Anticlockwise rotation is consistent with the large-scale shear of the FC (Figure 6).

Figure 10 depicts energy-preserving spectra such that the area under the curves is proportional to the variance. The low-frequency signal is not fully resolved owing to a chosen window length of 6.9 days. Degrees of freedom of the spectra increase with increasing $\omega$ from a minimum of 11.5 to 45.9.

The three dominant signals identified above, low frequency, 10 hour, and 27 hour, are visible to the eye to varying degrees in the time series shown in Figure 3. The low-frequency signal associated with variations of the FC stands out in the $v$ current component (Figure 3b). As indicated by the spectral analysis, the dominant higher frequency fluctuations in $v$ are a superposition of the 10- and 27-hour signals, which had similar strength. In contrast,
Figure 9. Subgrid-scale variability of (a) $u$ and (b) $v$ normalized by the velocity variance not contained in the largest spatial scales that were removed by bicubic spline fits to the data. Dominant period in hours of current variability for (c) $u$ and (d) $v$. The $e$-folding timescale in hours for (e) $u$ and (f) $v$.

Figure 10. Energy-preserving spectra of surface currents from OSCR at $\phi = +26.06^\circ$, east (dash-dotted line) and north (solid line) components for (a) $\lambda = -80.099^\circ$, (b) $\lambda = -80.074^\circ$, and (c) $\lambda = -80.034^\circ$, with distances from shore of 1.1, 3.6, and 7.6 km, respectively. Dotted lines in Figures 10a, 10b, and 10c indicate periods of 27 hours (inertial), 12.4 hours (semidiurnal tidal), and 10 hours, respectively.
5. The 10- and 27-Hour Signals

5.1. Methods

[32] Having identified the dominant signals in our current records, we now progress toward identifying the properties of the 10- and 27-hour fluctuations. In our analysis we utilize both spectral techniques and filtering in the time domain. The latter requires further explanation.

[33] The 10-hour signal and the 27-hour signal correspond to narrow, well-defined peaks in the spectra shown in Figure 10. This property invites using complex demodulation to extract the two signals and to further explore their time-space variability. Complex demodulation represents signals by means of slowly variable amplitude and phase. Phase information from different locations immediately allows inferences on signal propagation. Our approach uses routines from the “Starpac” mathematics package, which is based on work by Bloomfield [1976] [Donaldson and Tryon, 1986]. The properties of the complex demodulation are specified by carrier frequencies and low-pass 3-dB filter frequencies of 0.09/0.024 cph and 0.04/0.018 cph for the 10- and 27-hour signals, respectively. We thus account for the relative width of the 27-hour spectra peak and allow both the 10-hour signal and the semidiurnal tidal signal to pass into the demodulated 10-hour signal.

5.2. The 10-Hour Signal

[34] A look at the time series shown in Figure 3 suggests that the activity in the 10-hour band varied considerably over time. These variations are quantified in time-longitude contours of the amplitude of the demodulated 10-hour signal shown in Figure 11. The signal had pronounced short-term variability and a marked minimum of \( u \) amplitudes during year days 210–213, the period of the largest velocities and cross-shelf shear of the FC (see Figure 6). The largest 10-hour amplitudes in \( v \) of up to 0.6 m s\(^{-1}\) occurred after the latter event on year day 215. In this case, large \( v \) amplitudes extended to the eastern border of the OSCR domain, while \( v \) amplitudes otherwise peaked well inside the domain. Some high-amplitude events occurred across the OSCR domain, while others

![Image](image_url)
The amplitudes of the 10-hour signal were generally smaller than those of $v$ and there was a tendency for maxima in $u$ to appear farther offshore than those in $v$. The typical value of RMS amplitudes of the 10-hour signal was $0.2 - 0.3 \text{ m s}^{-1}$, as shown in the time averages of Figures 11b and 11d.

Figure 12. Along-shelf and cross-shelf phase propagation in $v$ and cross spectra of OSCR data. (a, b) Squared coherence between data at the center of the domain and at other latitudes, $\omega = 0.09375 \text{ cph}$ (10-hour signal, “O”) and $\omega = 0.0352 \text{ cph}$ (27-hour signal, “x”). (c, d) Cross-spectral phase of data at the center of the domain and at other latitudes marked as in Figures 12a and 12b. Time-average phases relative to the domain center from the complex demodulation of objectively analyzed OSCR data are added in Figure 12c as dots and crosses. Solid lines indicate linear fits to the latter data.

Figure 13. Current variance density and phase from cross spectra at (a) 10-hour period and (b) 27-hour period as function of depth, with data from the NE and SW moorings listed in Table 1. Current variance: $v$ component (thick lines, solid for NE and dash dotted for SW), $u$ component (thin lines, dashed for NE and dash dotted for SW); phase: “N” ($v$ component) and “n” ($u$ component) for NE, “S” ($v$ component) and “s” ($u$ component) for SW. Cross spectra were computed between data at 23.24 m depth (NE)/12.74 m depth (SW) and data across the water column.
Results from the two methods turned out to be highly consistent with each other. Given advection by the strong FC, one might expect northward signal propagation, and animations of the OSCR data indeed show features generally propagating northward. A formal analysis confirms this impression for the 10- and 27-hour periods. Figure 12c shows the phase of the $v$ component of the 10-hour signal relative to the center of the OSCR domain. The graph shows the phase at $\omega = 0.09375$ cph from cross spectra between the OSCR record at the center of the domain and records from varying latitude at a longitude of $\lambda = 80.06^\circ$. Figure 12c also shows time-average phases of the complex demodulated records relative to that of the domain center. A fit to these latter phases has a slope of about $-1300^\circ$ per degree of latitude. This is equivalent to a northward phase speed of 0.85 m s$^{-1}$ and a wavelength of 31 km. Note that this wavelength is substantially larger than the along-shelf width of the OSCR domain.

While we expected a northward signal propagation, we also found eastward propagation of the 10-hour signal, propagation away from the shore (Figure 12d). We discuss this finding in section 6 and associate it with typical properties of unstable, growing waves propagating along the shelf. Such instabilities have crests and troughs oriented at an angle toward the propagation direction; they “lean against the shear,” in meteorological terminology. Excluding data from east of $\lambda = 80.06^\circ$, where the coherence with the domain center is low (Figure 12b), the phase slope is about $-1350^\circ$ per degree of longitude, equivalent to a cross-shelf wavelength of $\sim 27$ km. The orientation of wave crests and troughs is thus $\sim 45^\circ$.

The northward and eastward phase propagation and the $45^\circ$ wave orientation in the 10-hour band are clearly visible in Figures 7g–7l, which show surface current maps extracted from the observations by means of complex demodulation and then remodulated. A comparison of the remodulated 10-hour currents in Figures 7g–7l with the full surface currents depicted in Figures 7a–7f allows the eye to pick out spatial signatures of the 10-hour signal in the observations. In contrast to its dominance in the time and frequency domains (Figures 2 and 10), the 10-hour signal does not stand out in the spatial domain. This is a result of it having relatively large spatial scales, wavelengths of $\sim 30$ km in $x$ and $y$, while the dominant spatial variability scales were much smaller, as found from the correlation analysis presented in section 4.1.

Further properties of the 10-hour signal concern current vector rotation and depth structure. Rotary spectra indicate dominant anticlockwise rotation at the 10-hour period (not shown). Such rotation is also clearly visible in Figures 7g–7l and is expected from the vorticity associated with the FC (section 3). The depth structure of the 10-hour signal can be examined using the “NE” and “SW” moorings located within the OSCR domain (Table 1). The signal variance decayed both toward the bottom and toward the surface, but the phase was approximately constant across the water column for the 10-hour signal, suggesting that the process was essentially barotropic (Figure 13a). Note that the larger $v$ component shows especially little depth variation of the phase, except near the bottom where signals became small. However, a cross-spectral analysis of current observations from the “Navy 150-m” mooring (Table 1) reveals a $180^\circ$ phase shift between near-surface and near-bottom $v$ at the 10-hour period. Hence the 10-hour signal appears barotropic in the shallow water of the shelf and just off the shelf break, but is clearly baroclinic farther offshore in deeper water.

5.3. The 27-Hour Signal

The space-time structure of the 27-hour signal showed some similarity to that of the 10-hour signal. Figure 14 again reveals marked short-term variability of $v$ amplitudes at the 27-hour period, with the largest $v$ also occurring after year day 215. The 27-hour signal remained energetic through the maximum of the largescale flow on year days 210–213 (Figure 6). Maximum 27-hour amplitudes occurred well within the OSCR domain, even during...
the year day 216 event. Typical RMS amplitudes were 0.2 m s\(^{-1}\), with decay toward the coast (Figure 14b).

The 27-hour signal showed northward signal propagation, except that measurements from north of \(\phi = 26.08^\circ\) behaved markedly different than the rest of the data (Figure 12c). We note that the statistical properties of the full surface current field changed in this region, as outlined in section 4.1 (see, e.g., Figure 9). Ignoring the region \(\phi > 26.08^\circ\), complex demodulation indicates a phase slope of \(-240^\circ\) per degree of latitude, or a wavelength of \(~170\) km. The corresponding phase speed is 1.7 m s\(^{-1}\). Again, the uncertainty in these estimates is quite large owing to the small size of the OSCR domain relative to the wavelength and to systematic variations in the phase slope as a function of latitude. The cross-shelf phase structure of the 27-hour signal is characterized by a decrease in relative phase west of the domain center and by an approximately constant phase east of it (Figure 12d).

As mentioned above, the 27-hour signal showed anticlockwise rotation with a very small \(u\) component such that the signal was almost linearly polarized. Cross-spectral analysis of the depth structure at the NE and SW mooring sites shows that the 27-hour signal also was essentially barotropic in the shallow near-coastal domain. Note that the phase of the dominant \(v\) component shown in Figure 13b is practically constant. Cross-spectra from the Navy 150-m mooring do not provide statistically significant phase information at the 27-hour period.

6. Summary and Discussion

We analyze the current variability of dominant 10- and 27-hour periods on and near the narrow and shallow shelf of Florida near Fort Lauderdale. The northward flowing Florida Current provides an ambient largescale vorticity of typically 4/ with marked variations associated with pulsations and/or meandering of the FC. The current variability showed three dominant signals, a low-frequency component associated with variations of the FC, a 27-hour signal, and a 10-hour signal. The 27-hour signal, having the local inertial period, had typical RMS amplitudes of 0.2 m s\(^{-1}\) with maxima of \(~0.5\) m s\(^{-1}\). It was nearly linearly polarized with anticlockwise rotation. Maximum amplitudes occurred beyond the shelf edge within the shoreward flank of the Florida Current and the Gulf Stream [Johns and Schott, 1987; Zantopp et al., 1987; Lee et al., 1995]. Similarity with submesoscale, near-inertial vortices found by Shay et al. [1998b] off the Florida Keys is also limited. As noted above, the 27-hour signal varies at the local inertial period and thus has a simple possible forcing or generating agent in the ubiquitous inertial oscillations of the ocean. In the high vorticity, nearshore environment examined here, circular and clockwise polarized motions cannot possibly survive, and we suspect that nonlinear processes transform the 27-hour signal to its observed linear polarization.

With respect to the 10-hour signal, we first argue that it cannot have been generated by the interaction of the barotropic semidiurnal tide with the spatially variable topography. Elsewhere on the continental shelves, the propagation of the barotropic tide past the shelf edge often creates internal semidiurnal tides as well as trains of large-amplitude, higher frequency internal waves. This mechanism cannot explain the 10-hour signal, however, for reasons of the most basic laws of linear wave kinematics. In an environment of time-invariant, spatially variable mean currents, the encounter frequency of any wave signal is conserved along rays [LeBlond and Mysak, 1978, equation 6.19b]. It is only the intrinsic frequency, measured by an observer moving with the mean current, that is changed by Doppler shifting. Relative to the Earth, any wave generated at the \(M_2\) or \(S_2\) tidal frequency keeps that frequency in the limit of linear dynamics. Nonlinear processes can, of course, transfer energy to other frequencies, but this has nothing to do with Doppler shifting.

The slanting crests of the 10-hour (and 27-hour) signal described in Section 5.2 are a telltale indicator of flow instability. The “leaning of the wave crests against the mean shear” is necessary if energy is to be transferred from the mean to the fluctuations [Gill, 1982, chapters 13.3–13.6]. This feature occurs both in 2-D and 3-D flow instability. In the course of our work we noticed the barotropic structure of the 10-hour signal near the shelf break and were thus led to suspect that the 10-hour signal was the result of barotropic instability of the zonally sheared FC. The mean current and vorticity profiles shown in Figures 6b and 6d indeed fulfill the Rayleigh criterion of 2-D flow instability [Holton, 1992, p. 255]: The \(\nu\) profile of \(\nu\) shown in Figure 6b changes the sign of its curvature within the domain. Phrased differently, the vorticity associated with the FC had a maximum at the shelf break (Figure 6f).

Barotropic flow instability problems more or less akin to our scenario have been investigated with respect to shelf waves by, for example, Niiler and Mysak [1971] and
with respect to the immediate vicinity of a beach by, for example, Bowen and Holman [1989]. The dynamics of such stable or unstable waves is summarized in the principle of conservation of potential vorticity, which is made up of a stretching term associated with the variable water depth $H(x)$, of the relative vorticity $V_z = \partial V_x / \partial x$ resulting from the cross-shore shear of the mean along-shore current $V(x)$, and of the relative vorticity $\zeta$ of the waves themselves:

$$\frac{d}{dt} \left( \frac{f + V_x + \zeta}{H} \right) = 0. \quad (4)$$

Here the dependencies $H = H(x)$ and $V = V(x)$ match our flow scenario. At the suggestion of one reviewer, we actually solved the linear barotropic instability problem based on equation (4) for the mean conditions in our observational domain. While the mean flow is indeed linearly unstable, the scales of the instabilities turned out to be incompatible with the 10-hour signal. Because of this negative result, we relegate an outline of how we solved the instability problem to Appendix A.

[47] In retrospect, the incompatibility of barotropic flow instability with the scales of the 10-hour signal is not surprising, even disregarding the fact that its depth structure was definitely baroclinic some distance east of the shelf break. The existence of barotropic instability depends on the overlap of the mean flow profile with the shallow shelf of some 25 m depth (Figures 6b, 6c, 6e, and 6f). The instability overlap of the mean flow profile with the shallow shelf of break. The existence of barotropic instability depends on the observational domain. While the mean flow is indeed linearly unstable, the scales of the instabilities turned out to be incompatible with the 10-hour signal. Because of this negative result, we relegate an outline of how we solved the instability problem to Appendix A.

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[48] We follow Bowen and Holman [1989] in setting up a linear stability problem related to the 4D observations. A volume transport stream function $\Psi$ is introduced as

$$-\Psi_y = Hu$$

$$\Psi_x = Hv,$$  \hspace{1cm} (A1)

where subscripts denote partial differentiation. The stream function is inserted in (4) and the resulting partial differential equation for $\Psi$ is linearized:

$$\left( \frac{\partial}{\partial t} + V_x \frac{\partial}{\partial x} \right) \left[ \frac{\Psi_y}{H} \right] - \left[ \frac{\Psi_x}{H} \right] = \Psi_x \left( \frac{V_x}{H} \right). \quad (A2)$$

Solutions are sought in the form $\Psi = \tilde{\Psi}(x) \exp(i(ky - \sigma t))$, where $\sigma = \sigma_r + i\sigma_i$ and $\Psi$ are complex and $k$ is the wave number along the $y$ axis. The problem is simplified by letting $H$ be constant and letting $V_x$ be constant in discrete intervals of $x$. Figure A1 depicts the representation of water depth, mean velocity, and potential vorticity chosen for the 4D situation. There are three intervals of constant shear, with $V_x \neq 0$ only in the middle band, and there are only two different water depths. This is the minimum configuration still able to represent the basic structure of water depth, velocity, and mean potential vorticity. In its linear form, and with $H = H(x)$, $V = V(x)$, and coast at $x = 0$, the Coriolis parameter $f$ does not enter the problem. Consequently, it is omitted from the estimate of vorticity shown in Figures A1a and 6f.

[49] With piecewise constant $H$ and $V_x$, the general solution to equation (A2) in each piece can be written as $\Psi(x) = a \sin(kx) + b \cosh(kx)$. Boundary conditions are $\Psi = 0$ at $x = 0$ and at $x = +\infty$. The stream function $\Psi$ and pressure, or, equivalently, the surface displacement $\bar{h} = -(gh)^{-1}[(V - c)\Psi_x - V\Psi]$ [see Bowen and Holman, 1989], have to be continuous in $x = [0, +\infty]$. Here $c = \sigma/k$ is the complex phase velocity. The matching conditions yield six equations with six unknowns. In order for nontrivial solutions to exist, the determinant of the coefficient matrix has to be zero, a condition that yields the dispersion relationship of the instability problem. In somewhat of a tour de
force, the problem can be solved analytically with the aid of the Mathematica program [Wolfram, 1999]. There are three roots, a stable mode and a pair of complex conjugate roots of which we seek the one with $\sigma_j > 0$. The expression for this root is characterized by stating that the functions sinh and cosh occur in it over 7000 times, each.

For the conditions shown in Figure A1, the most unstable wave has a north-south wavelength of $\sim 7$ km, a period of $\sim 17.5$ hours, and a northward phase speed of 0.1 m s$^{-1}$. The $\varepsilon$-folding timescale of the growing waves is 3.3 hours. These wave parameters clearly do not match the properties of either of the 10- or 27-hour signals. While detail of the setup depicted in Figure A1 could be debated, it is clear that the solution to the barotropic instability problem is qualitatively realistic and that there is no way to bend it toward matching the observed 10- or 27-hour signal. It is unclear if barotropic instability was important within the 4D domain. Figures 6a and 6d show that the properties of the low-passed flow changed continuously during the observations and indicate that barotropic instability conditions kept changing quantitatively and qualitatively.

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References