The 3D Oceanic Mixed Layer Response to Hurricane Gilbert

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ABSTRACT

Upper-ocean heat and mass budgets are examined from three snapshots of data acquired during and after the passage of Hurricane Gilbert in the western Gulf of Mexico. Measurements prior to storm passage indicated a warm core eddy in the region with velocities of $O(1)$ m s$^{-1}$. Based upon conservation of heat and mass, the three-dimensional mixed layer processes are quantified from the data. During and subsequent to hurricane passage, horizontal advection due to geostrophic velocities is significant in the eddy regime, suggesting that prestorm oceanic variability is important when background flows have the same magnitude as the mixed layer current response. Storm-induced near-inertial currents lead to large vertical advection magnitudes as they diverge from and converge toward the storm track. Surface fluxes, estimated by reducing flight-level winds to 10 m, indicate a maximum wind stress of 4.2 N m$^{-2}$ and a heat flux of 1200 W m$^{-2}$ in the directly forced region. The upward heat flux after the passage of the storm has a maximum of 200 W m$^{-2}$ corresponding to a less than 7 m s$^{-1}$ wind speed.

Entrainment mixing across the mixed layer base is estimated using three bulk entrainment closure schemes that differ in their physical basis of parameterization. Entrainment remains the dominant mechanism in controlling the heat and mass budgets irrespective of the scheme. Depending on the magnitudes of friction velocity, surface fluxes and/or shear across the mixed layer base, the pattern and location of maximum entrainment rates differ in the directly forced region. While the general area of maximum entrainment is in the right-rear quadrant of the storm, shear-induced entrainment scheme predicts a narrow region of cooling compared to the stress-induced mixing scheme and observed SST decreases. After the storm passage, the maximum contribution to the mixed layer dynamics is associated with shear-induced entrainment mixing forced by near-inertial motions up to the third day as indicated by bulk Richardson numbers that remained below criticality. Thus, entrainment based on a combination of surface fluxes, friction velocity and shear across the entrainment zone may be more relevant for three-dimensional ocean response studies.

1. Introduction

The mutual response of the coupled hurricane–ocean system is one of the more extreme air–sea interaction events. As the underlying boundary of a translating hurricane, the role of the upper ocean in formation and maintenance of storms has long been recognized (Palmen 1948) and sea surface temperature (SST) has been shown to have a direct correlation with wind speeds (DeMaria and Kaplan 1994). However, during the storm passage, the oceanic mixed layer is well mixed, and it is the mixed layer temperature that controls the latent and sensible heat fluxes to the atmosphere. This is further complicated by preexisting oceanic variability such as Gulf Stream system and mesoscale eddies that modulate the upper ocean heat, mass, and momentum balance due to horizontal advection.

Early observations of ocean response during hurricane passage included ship-based (e.g., Leipper 1967; Fedorov et al. 1979) and aircraft-based (Black 1983) temperature measurements and fortuitous mooring measurements of currents deployed in support of other experiments (Brooks 1983; Shay and Elsberry 1987). Temperature observations suggested a well-defined modulation of SST by the storm parallel to the track with maximum SST decreases of $1^\circ$ to $6^\circ$C to the right of the track. Based on 10 years of airborne expendable bathythermograph (AXBT) data, Black (1983) found a general crescent-shaped pattern of SST decreases in the right-rear quadrant of the directly forced region. This
rightward bias of SST reduction was attributed to the nonlinear mixed layer current response and to the stronger winds located on the right side of the storm track due to storm motion (Chang and Anthes 1978; Price 1981). Estimation of boundary layer fluxes over the storm forcing periods, revealed that the heat loss to the atmosphere contributed only 20% or less of the observed SST decreases consistent with the numerical simulations (Elsberry et al. 1976; Price 1981). However, assumed quiescent initial conditions in these numerical models do not reflect realistic conditions in either a mesoscale eddy field or oceanic frontal regime.

Moored current measurements have provided the temporal evolution at fixed vertical levels; yet the oceanic momentum response, and in particular the ocean current shear and horizontal advection, could not be resolved because of the limited spatial sampling. To resolve these issues, a series of airborne expendable current profilers (AXCPs) were successfully deployed from National Oceanic and Atmospheric Administration (NOAA) WP-3D aircraft during three hurricanes, Norbert (1984), Josephine (1984), and Gloria (1985), and provided a three-dimensional description of ocean currents within the directly forced region for the first time (Sanford et al. 1987). Observed maximum velocities ranged from 0.8 to 1.7 m s$^{-1}$ and were found on the right side of the storm track. However, as measurements were made only during the storm, the evolving three-dimensional internal wave wake and the associated mixed layer response were not resolved by the measurement strategy. In addition, the AXCP data acquired during Josephine and Gloria were inadequate to resolve preexisting oceanic variability of cooler sea surface temperatures and large thermocline currents (Price et al. 1994) associated with the subtropical convergence zone (Voorhis 1969; Black et al. 1988). Quantifying the changes in upper ocean balances due to such preexisting conditions is important not only within the context of air–sea heat and momentum exchange between a hurricane and the ocean, but also this variability modifies the oceanic response to the transient forcing.

To examine the evolving three-dimensional ocean response and the near-inertial internal wake, an experiment was conducted in the western Gulf of Mexico during hurricane Gilbert (Fig. 1) from NOAA WP-3D aircraft by deploying 76 AXCPs and 51 AXBTs prior to, during, and subsequent to the passage of the storm (Shay et al. 1992, hereafter S92). Prior to this experiment, the trajectory of a drifting buoy indicated the presence of a large warm-core eddy northeast of the forecast track and a mobile sampling strategy was adapted to resolve the preexisting oceanic current variability as described in S92. The derived mixed layer depths (MLDs) and SSTs during the four experimental periods, denoted as Prestorm, Storm, Wake 1, and Wake 2, are shown in Fig. 2. Prestorm MLD of a nearly uniform 30 to 40 m in the experimental domain increased by about 5 to 30 m associated with a 3.5°C SST or mixed layer temperature decrease. Moreover, Prestorm SST distribution did not indicate the presence of the Loop Current warm core eddy (LCWCE), which is typical in the Gulf of Mexico during summer. These LCWCE are characterized by a markedly different temperature–salinity ($T$–$S$) relationship of subtropical origin with strong anticyclonically rotating currents in the upper layers. Thus, to examine the oceanic response to hurricane Gilbert, the geostrophically balanced currents were removed from the current profiles by assuming a thermal wind balance. The linear near-inertial response dominated the mixed layer momentum response including energetic near-inertial shears across the entrainment zone (Shay et al. 1998, hereafter S98).

In this paper, the heat and mass balances in the oceanic mixed layer excited by the passage of Hurricane Gilbert are investigated in the presence of preexisting oceanic variability. The approach uses mixed layer conservation arguments to assess the 3D mixed layer heat and mass budgets from the observed fields. This paper is organized as follows: In section 2, diagnostic mixed layer models are reviewed including the data resources and turbulent closure schemes that are used to determine the relative magnitudes of advection due to prestorm velocities and vertical mixing at the mixed layer base. These entrainment velocity closure schemes are based on Pollard et al. (1973, hereafter PRT), Kraus and Turner (1967, hereafter KT) and Deardorff (1983) and use the observed data and derived fluxes. In section 3, the results are discussed and the dominant physical mechanisms that contribute to the mixed layer heat and mass budgets are delineated from the three data snapshots. Results are summarized with concluding remarks and future measurement strategies in section 4.

2. Methodology and data resources
a. Diagnostic mixed layer model

Analysis of SSTs and MLDs not only indicates a regime of mixed layer cooling and deepening but that the potential exists for strong advection due to prestorm oceanic variability in the vicinity of LCWCE (S98). To understand the effects of LCWCE on the mixed layer heat and mass balance, bulk mixed layer expressions based on conservation laws (Elsberry et al. 1976) are used in this study. Estimation of each of the terms in these expressions using the observed and derived quantities delineates their relative importance on mixed layer cooling and deepening processes during and after the passage of Gilbert. The conservation of heat in the mixed layer is given by the equation

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( u \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( v \frac{\partial T}{\partial y} \right) - \frac{Q_e}{\rho_0 c_p h} - Y \frac{\delta e}{h} \Delta T, \tag{1}$$

where $T$ is defined as the bulk mixed layer temperature

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( u \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( v \frac{\partial T}{\partial y} \right) - \frac{Q_e}{\rho_0 c_p h} - Y \frac{\delta e}{h} \Delta T, \tag{1}$$
and the sea surface (thereby neglecting microstructure effects), \((u, v)\) are bulk mixed layer horizontal velocities, \(Q_0\) is the surface heat flux (latent and sensible), \(w_e\) is the entrainment velocity, \(\Delta T\) is the temperature difference between mixed layer and thermocline, and the step function \(\gamma = 0\) if \(w_e > 0\), 1 otherwise in accord with the second law of thermodynamics. The corresponding conservation of mass (or thickness) in the mixed layer is

\[
\frac{\partial h}{\partial t} = -u \frac{\partial h}{\partial x} - v \frac{\partial h}{\partial y} - w_{-h} + w_e, \tag{2}
\]

where \(h\) is the mixed layer depth and \(w_{-h}\) is the baroclinic vertical velocity at the mixed layer base.

Spatial distribution of quantities \(T\) and \(h\), derived from the AXBT and AXCP temperature measurements, are shown in Fig. 2. Estimation of surface heat fluxes from the oceanic mixed layer requires the atmospheric boundary layer wind and thermodynamic quantities. Entrainment at the mixed layer base is quantified from observed quantities using various closure schemes. By including advective terms, \([-u(\partial T/\partial x) - v(\partial T/\partial y)]\) and \([-u(\partial h/\partial x) - v(\partial h/\partial y)]\) in these equations, the relative importance of preexisting currents in the heat and mass budgets is estimated by partitioning the observed mixed layer currents into storm-induced and preexisting components.
which would indicate their possible influence on the oceanic feedback to a hurricane.

b. Boundary layer wind field

Hurricane Gilbert was one of the strongest storms in the Atlantic basin in recent history. Over the warm waters of the Western Caribbean sea, the storm explosively strengthened to Category 5 with a minimum central pressure of a record 888 mb and flight-level winds exceeding 80 m s⁻¹ at a radius of about 15 km from the center prior to crossing the islands of Cancun and Cozumel. The storm translated at a speed of 7 m s⁻¹ and winds greater than 32 m s⁻¹ extended beyond 100 km from the eye. After crossing the Yucatan Peninsula, the translation speed decreased to 5.6 m s⁻¹ as a weakened Gilbert moved over the Gulf of Mexico.

During the upper ocean response experiment, the aircraft acquired wind and thermodynamic measurements at a flight-level of 850 mb that indicated a maximum 10-min sustained winds of about 53 m s⁻¹. These flight-level winds are reduced to 10-m standard height using...
the boundary layer model of Powell (1980) for two-time periods (Fig. 3). Reduced flight-level winds are combined with the European Centre for Medium-Range Weather Forecasts (ECMWF) model-generated 10-m wind field and buoy measurements. These data are objectively analyzed by a method in which cubic B-splines minimize the deviations between the input observations and analysis output (Powell and Houston 1996; Franklin et al. 1993; Ooyama 1987) to obtain boundary layer winds over the Gulf of Mexico at 0600 UTC 16 September 1988 (Fig. 4). The analyzed wind field is broad with wind speeds up to 30 m s$^{-1}$ extending out to 160 km from the eye, and the maximum sustained 10-min wind is about 40 m s$^{-1}$. As shown in Figs. 3 and 4, winds at the secondary $R_{\text{max}}$ exceeded the primary wind maximum and this secondary eyewall convection is also apparent in the airborne radar data as well as Special Sensor Microwave/Imager data (not shown). This broad wind structure with dual maxima will have an impact upon the surface flux because of its wind speed dependency.

c. Surface fluxes

In an oceanic surface mixed layer, turbulent transfer of mass, heat, and momentum occurs at the air–sea interface and across the base of the layer in the entrainment zone. Air–sea exchanges of heat and momentum and their spatial distribution are estimated using bulk aerodynamic formulas:

\[
\tau = C_D U_{10} U_{10} \rho_a,
\]

\[
Q_s = C_H U_{10} (T_{10} - T_s) \rho_a C_p,
\]

\[
Q_L = C_L U_{10} (q_{ss} - q_{10}) \rho_a L_v,
\]

where $\tau = \tau_i + \tau_j$ is the wind stress vector, assumed to be aligned along the surface wind vector at 10 m ($U_{10} = |U_{10}|$), $Q_s$ is the sensible heat flux, $T_{10}$ is the air temperature at 10 m, $T_s$ is the air temperature at the sea surface assumed to be the SST, $Q_L$ is the latent heat flux, $q_{10}$ is the specific humidity of air at 10-m height, and $q_{ss}$ is the specific humidity at the sea surface assuming saturation at a given SST.
Fig. 4. (a: top) Surface winds derived from flight level reduced, ECMWF surface and buoy winds for 0600 UTC 16 Sep 1988. Every eighth data point from the analyzed field is plotted as a barb (in knots) with contours representing the wind magnitudes in m s$^{-1}$. (b: lower left) Total surface heat flux (latent and sensible) derived using the analyzed winds in the experimental domain for 0600 UTC 16 Sep 1988 (Storm). Due to the broad wind structure, total heat flux exceeded 1000 W m$^{-2}$ upto $3R_{max}$ from the track.
The surface drag coefficient \( (C_D) \) was computed using the wind-speed-dependent formulation of Large and Pond (1981), that is, \( C_D = 1.14 \times 10^{-3} \) for \( U_1 \leq 10 \) m s\(^{-1}\) or \((0.49 + 0.065U_1) \times 10^{-3} \) for \( U_1 > 10 \) m s\(^{-1}\). Though this formulation is not valid beyond 25 m s\(^{-1}\), extrapolated \( C_D \) to 40 m s\(^{-1}\) introduces negligible uncertainty from the values estimated using the WAMDI Group (1988) formulation. Note that considerable uncertainty exists in the surface drag coefficient at very high wind speeds and leveling off of \( C_D \) beyond a certain wind speed have been suggested based on atmospheric budget studies (Frank 1985) and WKBJ-scaled internal wave fluxes derived from the Gilbert current profiles (Shay 1997). In addition, the above formulation assumes a fully developed sea whereas in a hurricane, the sea is almost always under developed because of the short fetch due to the wind field curvature and duration of strong winds. Constant values of \( 1 \times 10^{-3} \) for the sensible heat flux coefficient \( (C_{H}) \) and \( 1.2 \times 10^{-3} \) for the latent heat flux coefficient \( (C_{L}) \) are used because of the near-neutrality of the atmospheric boundary layer in hurricanes. Parameters \( \rho, C_p \) and \( L \) represent density of air, heat capacity of air, and latent heat of vaporization, respectively.

Using boundary layer winds and the bulk formulas, the wind stress \( (\tau) \) and the friction velocity \( (u_f) \) and their spatial distribution in the directly forced region are estimated. The maximum wind stress was 4.2 N m\(^{-2}\) with the friction velocity of about 2.0 m s\(^{-1}\). Assuming a constant air–sea temperature and humidity difference, total heat flux \( (Q_0) \) is obtained as the sum of latent and sensible heat fluxes (Fig. 4b). Because of the wind speed dependency in the bulk formulas, the spatial distribution of \( Q_0 \) follows wind field distribution with a maximum of 1200 W m\(^{-2}\). While similar values were estimated in the boundary layer of Tropical Cyclone Kerry (Black and Holland 1995), coupled ocean–atmosphere simulations of Hurricane Opal (1995) suggested larger values of the surface heat fluxes over the Gulf of Mexico (Hong et al. 2000). Radiative fluxes are small during the storm passage and heat flux due to precipitation is not included here as the error involved is less than 10% of the total heat flux as indicated by the estimates using historical rainfall values in a hurricane (Miller 1958; Black 1983). During the relaxation stage (Wake 1 and Wake 2), surface fluxes are estimated using the ECMWF model boundary layer fields.

d. Entrainment fluxes

Turbulent transfer across the mixed layer base is represented using the entrainment velocity closure from the three snapshots of velocity measurements. Based upon the observational analyses of Black (1983), entrainment velocity parameterizations used in previous numerical studies simulated the magnitude of cooling, yet could not reproduce the crescent-shaped SST cooling pattern. This may either be due to an entrainment formulation that is not suitable for all of the directly forced region or that the momentum response is not simulated correctly by the model. To examine the dominant mechanisms, entrainment velocities at the mixed layer base are estimated using three entrainment velocity parameterizations. First, PRT and Price (1981) specified the following relationship between entrainment velocity and bulk Richardson number \( (R_b) \) based upon vertical current shear

\[ w_e = 5 \times 10^{-4} R_b^{3/4} \delta V, \]

where the bulk Richardson number, \( R_b \), is estimated from the expression

\[ R_b = \frac{gah\Delta T}{\delta V^2}, \]

where \( g \) is the acceleration due to gravity, \( \alpha \) is the coefficient of thermal expansion, \( h \) is the mixing length scale, and \( \delta V \) is the difference between the depth-integrated mixed layer and the upper thermocline velocity. In regions where the \( R_b \) is greater than 1, \( w_e \), associated with shear instabilities is set to zero. The bulk Richardson numbers ranged from 0.2 to 5 during the experiment, with the lowest values found in the high-shear regimes. Assuming a 5% uncertainty in the \( \delta V \) values, uncertainty in \( R_b \) is approximately 10%. However, because of the high nonlinearity in the \( w_e \) expression, this introduces uncertainties by as much as 50% in the entrainment velocity. It should be noted here that, for larger shear values, this uncertainty is reduced significantly because of the increased signal to noise ratio.

A second approach is based upon the formulation of Kraus and Turner (1967) in which the entrainment velocity is specified based on stress-induced turbulence and surface buoyancy flux as

\[ w_e = \frac{1}{\Delta T} \left( \frac{c_1 u_f + c_2 Q_0}{\rho C_p} \right). \]

Empirical mixing coefficients, \( c_1 \) and \( c_2 \) in the above expression set to 2.5 and 0.4, respectively, are similar to those used in ocean response model studies (Elsberry et al. 1976; Chang and Anthes 1978). However, Huang (1986) documented the variability in these mixing coefficients and suggested that surface waves may also play a role in the mixing process that may modify these coefficients. In the KT scheme, entrainment velocity prediction is instantaneous with the wind, whereas the shear-induced \( (\delta V) \) schemes represent an integrated effect because strong shears at the mixed layer base depend on the time history of wind stress and near-inertial response.

Deardorff (1983) developed an entrainment scheme based on the turbulent kinetic energy (TKE) equation and second-moment equations for the buoyancy flux and the density perturbation across the entrainment zone. Using various functions and empirical constants from experimental results, the TKE budget was closed using
Richardson numbers: $R_i = c_i^2 u_i^2; R_w = c_i^2 w_i^2; \text{ and } R_s = c_i^2 \Delta \gamma^2$ where $c_i = \sqrt{g\alpha \Delta T}$ and $w_i$ represents the convective velocity scale

$$w_i^3 = \frac{Q_y h}{\rho_y C_p}.$$ 

These functions represent stress-induced, convection-induced, and shear-induced turbulent mixing parameterizations, respectively. Following Deardorff, $R_w$ and $w_i$ are defined as a combination of $R_i, R_s,$ and $w_i$ as

$$R_i^* = (R_i^{3/2} + \eta^2 R_s^{3/2})^{-2/3} \text{ and } w_i^* = (w_i^{1/3} + \eta u_r^{1/3}).$$

where $\eta$ is the mixing efficiency of the wind stress that has a maximum value of 1.8. The normalized entrainment rates $w_i/w_i^*$ estimated at the AXCP locations from Storm, Wake 1, and Wake 2 data are shown superposed on Fig. 3 from Deardorff (1983) in Fig. 5. During the Storm experiment (Fig. 5a), high wind speeds and large current shears at the mixed layer base force all three Richardson numbers to decrease implying that all three processes were important during the Storm. By contrast, relatively low winds lead to higher $R_s$ and $w_i$ during Wake 1 and 2 experiments (Figs. 5b,c) whereas the $R_i$ values remained low during low winds during Wake 2. That is, strong current shears that were set up during the Storm continued during Wake 1 as the mixed layer depths reached their maximum values. By the time of the Wake 2 experiment, the $w_i$ was large over most of the domain, which precluded any additional layer deepening induced by velocity shear (S92). It should be noted here, while the PRT scheme uses a critical bulk Richardson number criteria to initiate mixing, such a cutoff is not defined in the Deardorff scheme. Given their relative simplicity and direct relationship with the measured quantities in these three experiments, these closure schemes are used here to examine the mixed layer heat and mass budgets.

e. Advective velocities

1) Geostrophic velocities

Prestorm velocities in the experimental domain are predominantly due to the presence of LCWCE during the passage of Hurricane Gilbert. To delineate the effects of prestorm oceanic variability, observed velocity components are expressed as

$$u = u_x + u_r + u_*,$$  \hspace{1cm} (6)

$$v = v_x + v_r + v_*,$$ \hspace{1cm} (7)

where $u, v$ are the AXCP observed velocities less surface-wave orbital velocities, $u_r, v_r$ are geostrophic velocities, $u_*, v_*$ are the near-inertial velocities and $u, v$ are the residual velocities. The eddy was extensively sampled two months before the storm and a near-complete CTD dataset was acquired one month after the storm passage (SAIC 1989). This and other climatological data revealed distinct $T$--$S$ relationships of the LCWCE and gulf common waters. Using these $T$--$S$ relationships, dynamic heights are derived from temperature observations to estimate geostrophic velocity components in the experimental domain. In the LCWCE, peak velocities of 1 m s$^{-1}$ with dynamic height $(\psi)$ of 1.3 dyn m are estimated from the data (S98) with respect to a level of no motion of 750 m using the following equations:

$$u_r = -\frac{g}{f} \frac{\partial \psi}{\partial y} \text{ and } \hspace{1cm} (8)$$

$$v_r = \frac{g}{f} \frac{\partial \psi}{\partial x}. \hspace{1cm} (9)$$

Maximum geostrophic velocities in the mixed layer for Storm, Wake 1, and Wake 2 experiments are on the order of 1 m s$^{-1}$ in the eddy with the eddy center located near 5RT to the right of the storm track (Fig. 6). The spatial velocity structure during Storm differs from those of Wake 1 and Wake 2. Note that $u_r$ and $v_r$ may also include velocities generated by geostrophic adjustment due to storm passage. Quantitatively, for a fast moving storm such as Gilbert, 5% to 20% of these velocities are due to the passage of the storm and the isopycnal displacement due to the adjustment process (Rossby 1938; Veronis 1956). Close to the track (0 to 3 RT), storm-induced geostrophic currents are 0.1 to 0.2 m s$^{-1}$ as part of the adjustment to transient forcing (S98).

2) Near-inertial velocities

Based on the air–sea parameters, the ocean response due to Hurricane Gilbert was expected to be predominantly baroclinic, with strong near-inertial wave excitation. However, because of the strong background velocity structure of LCWCE, the estimated geostrophic velocities were removed from the measured velocities to delineate the hurricane-induced near-inertial response. Gilbert was moving in a nearly straight track at 290°T with an average speed of 5.6 m s$^{-1}$ enabling the use of a storm-centered coordinate system translating at the velocity $U_0$ of the storm (Geisler 1970; Shay et al. 1989; Price et al. 1994). By fixing a time, $t_e$ corresponding to the eye of the storm ($X_e = 0$), an AXCP deployed at time, $t_d = t_e + \Delta t$ and geographic position, $X_d = X_e + \Delta X$ is transformed to $X_d = X_e - X_e - \Delta t U_0$ in the storm coordinate system. For a steadily moving storm, along-track distance $X_e$ is then converted to time, $t_d = X_e/U_0$. Using this scheme, the AXCPs deployed during the Storm, Wake 1, and Wake 2 experiments were placed at different time/distance along the wake facilitating the investigation of the evolving three-dimen-
Fig. 5. Normalized entrainment rate at each AXCP location estimated from the data and three Richardson numbers superposed on Fig. 3 from Deardorff (1983). The $x$ axis is $R_s = (R_s^{3/2} + \eta R_s^{3/2})^{-3/2}$ and $y$ axis is $w_e/\nu_s^*$, $w_e = (w_s^* + \eta w_s^*)^{-1/2}$. The solid lines are drawn for fixed bulk Richardson number indicated: (a) Storm, (b) Wake 1, and (c) Wake 2. Note the increasing $R_s$ from Storm to Wake 2.
Based on the near-inertial model of Rossby and Sanford (1976) and using a decay scale of four inertial periods for the forced near-inertial motion in the wake of a hurricane (Price 1981; Gill 1984), the near-inertial velocity components in the mixed layer are written as

\[ u(t) = A_u \cos(\sigma t + \phi_u) e^{-\omega t} + u_s, \]  
\[ v(t) = A_v \sin(\sigma t + \phi_v) e^{-\omega t} + v_s, \]

where \( e^{-\omega t} \) represents the decay of near-inertial currents in the wake of a storm. These equations are fit to the observed residual currents \( u - u_s \) and \( v - v_s \) using the Levenberg–Marquardt technique (Marquardt 1963) and the values of \( A_u, A_v, \phi_u, \) and \( \phi_v \) are estimated by minimizing the residual covariance \( \langle u,v \rangle \) for trial frequencies \( \sigma \) between 0.8 \( f \) and 1.2 \( f \). These fits were accurate to within an rms error of 15 cm s\(^{-1}\) in the mixed layer and a few centimeters per second in the thermocline (S98).

During the Storm experiment (Fig. 7a), divergent near-inertial velocities from the track were about 1.2 m s\(^{-1}\) that reduced to 1 m s\(^{-1}\) during Wake 1 as velocities started to converge during the next inertial cycle associated with the downwelling of the isotherms in the thermocline (Fig. 7b). During Wake 2, estimated near-inertial velocities were on the order of 0.7 m s\(^{-1}\) compared to the observed maximum of 0.80 m s\(^{-1}\), sug-
suggesting the assumed $e$-folding scale of 4 IP is reasonable. These velocities are used to estimate the near-inertial advection of heat and mass in the mixed layer.

3) **Vertical velocities**

Vertical velocity at the mixed layer base is estimated by computing the horizontal divergence from these near-inertial velocities. However, the estimates and patterns of the vertical velocities had a strong dependence on the spline parameters used in the objective analysis (OA) scheme because of uncertainties in estimating divergence. Shay et al. (1998) have shown (see Figs. 16 and 23 of S98) that the near-inertial horizontal velocities estimated using the Rossby–Sanford scheme are highly correlated with the forced analytical model velocities of Shay et al. (1989). Hence, the vertical velocities at the mixed layer base from the analytical model are used to estimate the magnitude of vertical advection.

**f. Uncertainty estimates**

The temperatures and currents in the upper ocean measured by AXBTs and AXCPs are subject to measurement uncertainties (Sanford et al. 1982; Gregg et al. 1986) that result in uncertainty in the estimated mixed layer quantities. Assuming that the variables are independent of each other, uncertainty limits for advec-
tive, surface and entrainment flux terms in the heat and mass budgets are estimated as a percentage in an rms sense. Illustrating this with the surface flux estimate \( Q_o \), it is assumed that the variables \( \Delta T_w = T_w - T_{io} \) and \( \Delta q_w = q_w - q_{io} \) are independent and the uncertainty in \( Q_o \) is obtained from

\[
\sigma_{Q_o}^2 = (C_{H\rho}\rho C_w \Delta T_w + C_{H\rho}\rho C_{uw} \Delta q_w)^2 \sigma_{T_w}^2 + (C_{H\rho}\rho C_{uw} U_{10})^2 \sigma_{q_{uw}}^2 + (C_{H\rho}\rho C_{uw} U_{10})^2 \sigma_{q_{uw}}^2,
\]

where appropriate values for \( \sigma_{T_w} \) and \( \sigma_{q_{uw}} \) are used. A similar approach is used to estimate uncertainties for individual terms in the mixed layer budget that are listed in Table 1. The maximum and minimum uncertainty represents the range for the field during Storm, Wake 1, and Wake 2 snapshots and in general less uncertain values correspond to the Storm experiment. The surface flux terms have the least uncertainty and the PRT entrainment flux terms have the highest uncertainty that correspond to Wake 2. Thus, the measurement uncertainties do not significantly affect the signal to noise ratio. The uncertainties involved in OA mapping of the data fields are discussed in detail in S92 (see Fig. 4 of S92). A normalized error value of 0.6 is used to estimate the mapping uncertainty limits listed in Table 1 as it defines the observational domain.

3. Mixed layer balance

a. Heat budget

1) Forced stage—Storm

The contribution from advective, surface and entrainment fluxes to the heat balance from (1) are estimated using the analyzed mixed layer velocity, temperature, surface flux, and entrainment fields in the directly forced regime. Mixed layer temperature (Fig. 2b, first column) started decreasing from Prestorm values ahead of the storm, and significant cooling near the track introduces strong temperature gradients. These gradients are computed using a bicubic spline fitting procedure, which has the advantage of providing smooth estimates of first and second derivatives that are continuous at every point in comparison to a finite differencing scheme (Inoue 1986).

The geostrophic advection term \((U_\perp \cdot \nabla T)\) has a maximum value of \(-0.69^\circ C \, d^{-1}\) in the eddy region at a distance of 4 to 5 \( R_{max} \) from the storm track. Because of the positive gradients between the track and the eddy, prestorm anticyclonic geostrophic velocities transport cooler water from near the storm track toward the eddy region in the front half of the storm and the warmer eddy water toward the track in the rear half (Fig. 8a).

Near-inertial advection has maximum values close to the storm track because of the larger near-inertial velocities (Fig. 8b). The maximum cooling rate due to surface heat loss to the atmosphere is \(-0.69^\circ C \, d^{-1}\). The pattern of this surface heat loss follows the wind speed due to wind speed dependency in the bulk aerodynamic formula and the assumed constant air–sea temperature and humidity difference (Fig. 8c). The important result here is that geostrophic advection is as large as the surface flux term in the overall heat budget. Thus, prestorm current features are important in understanding upper ocean response and coupled ocean–atmosphere interactions during hurricane passage.

However, the maximum observed mixed layer cooling is due to entrainment heat flux. The local rate of temperature change due to entrainment ranged from \(-17 \text{ to } -30^\circ C \, d^{-1}\) that mainly depended on the entrainment velocity \( w_e \). The local maxima tend to be enhanced in the KT entrainment flux term corresponding to the local wind maxima (Fig. 4a). The entrainment flux predicted by PRT scheme (Fig. 8d) has negligible entrainment in the eddy region where geostrophic advection dominates the heat balance. Both the KT (Fig. 8e) and Deardorff (Fig. 8f) schemes predict significant entrainment heat flux \((-4^\circ C \, d^{-1})\) away from the track due to Gilbert's broad wind field. While the geostrophic advection is quasi steady over storm forcing scales, entrainment must be intermittent at this rate to produce the observed mixed layer cooling. Despite comparable maxima of entrainment flux by the three schemes, the mixed layer (ML) budget differs significantly as indicated by the average rate of cooling over the experimental domain due to entrainment fluxes of PRT, KT, and Deardorff schemes of \(-2.4^\circ C \, d^{-1}, -5.3^\circ C \, d^{-1}, \text{ and } -6.2^\circ C \, d^{-1}\), respectively. Depending upon the particular entrainment scheme used to quantify the heat flux, the geostrophic advection contributed between 10% and 80% to the total ML heat budget in the LCWCE region although the surface heat fluxes are unaffected because of the lack of SST signature of the eddy in the domain.
FIG. 8. Heat budget during the Storm experiment in °C d⁻¹: (a) geostrophic advection, (b) near-inertial advection, (c) surface flux term, (d) entrainment flux (KT) (e) entrainment flux (PRT), and (f) entrainment flux (Deardorff). The lower color scale is for (d), (e), and (f).
fluxes are assumed, using KT entrainment formulation for simplicity with no advection requires

\[ \frac{dh}{dt} \propto \frac{u^*_h}{h g \alpha (T - T_b)}, \]

where \( T_b \) is the temperature at the mixed layer base. The implicit assumption here is that the temperature is representative of density. For profile 215 in the eddy (Fig. 9c), \( T_b \) is almost constant up to 150 m resulting in \( \Delta T = T - T_b \), whereas for profile 209 or 213 (Figs. 9a,b) outside the eddy, \( \Delta T = T - T_b + \Gamma z \), where \( z = h - H \), \( H \) being the initial mixed layer depth and \( \Gamma \) the temperature lapse rate. Thus, for the same initial \( h \) and \( \Delta T \), it is obvious from the above expression that ML deepening rate for profile 215 will be higher because of less initial stability. However, this does not translate into a larger cooling rate in the eddy. Neglecting surface fluxes and advective terms, it is seen from (1) that \( dT/dt \propto h^{-2} \) and thus the rate of cooling for a larger \( h \) in the eddy will be less than that in a noneddy background for the same initial temperature. Since there is a significant background velocity in the case of LCWCE, a prestorm thermal structure \( [T(z)] \) may be advected into the directly forced region, resulting in negligible SST reduction. Thus, deeper, warmer layers provide a nearly continuous source of heat to a storm for possible intensity changes as observed during Hurricane Opal through enhanced heat exchanges with the atmosphere (Shay et al. 2000).

2) RELAXATION STAGE—WAKE 1

The mixed layer temperature field during Wake 1 indicates water of 25.5°C located on the right side of the track between 0 to 2 \( R_{max} \) and increased in the cross-track direction toward the LCWCE regime to more than 27°C. Due to these substantial cross-track temperature gradients, the geostrophic advection term has a maximum value of \(-0.43 \text{°C d}^{-1}\) in the eddy region (Fig. 10a). By contrast, the near-inertial advection terms are \(1.3 \text{°C d}^{-1}\) close to the storm track and \(-1\) to \(0.5 \text{°C d}^{-1}\) between 2 and 3 \( R_{max} \) (Fig. 10b). The implication here is that the eddy continued to modulate the ML heat budget. The corresponding surface heat flux estimated from the ECMWF-derived boundary layer fields indicates a maximum of 214 W m\(^{-2}\), which leads to a negligible rate of surface flux-induced cooling of about 0.1°C d\(^{-1}\) (Fig. 10c). The rate of ML cooling due to entrainment heat flux predicted by KT scheme is small with values of 0.17°C d\(^{-1}\) (Fig. 10d) whereas the PRT
Fig. 10. Same as Fig. 8 during the Wake 1 experiment: (a) geostrophic advection, (b) Near-inertial advection, (c) surface flux term, (d) entrainment flux (KT 67), (e) entrainment flux (PRT), and (f) entrainment flux (Deardorff).
and Deardorff schemes suggest a maximum cooling rate of more than $8.64^\circ C \, d^{-1}$ (Figs. 10e,f) near the track that corresponds to the cold wake found along the track (Fig. 2c). Thus, continued cooling in the wake due to shear instabilities is an order of magnitude larger than the rate predicted by the KT scheme. Clearly, PRT and Deardorff schemes are more appropriate for relaxation stage of the thermal response given the high current shears induced by the forcing.

3) Relaxation Stage—Wake 2

The Wake 2 SST snapshot (Fig. 2d) indicates an eastward displacement of the cooler pool of $25.5^\circ C$ water from the Wake 1 location. During Wake 2, estimated geostrophic advection term has magnitudes ranging from $-0.43^\circ C \, d^{-1}$ to $0.85^\circ C \, d^{-1}$ (Fig. 11a). The near-inertial advection terms ranged from $0.6^\circ C \, d^{-1}$ near the storm track to $0.43^\circ C \, d^{-1}$ around 2 to 3 $R_{max}$ to the right of the storm track (Fig. 11b). Estimated surface heat fluxes have a maximum of 110 W m$^{-2}$, and therefore surface fluxes induce mixed layer cooling at a rate less than $0.04^\circ C \, d^{-1}$ (not shown). As in the Wake 1 experiment, rate of cooling due to entrainment predicted by KT is less than $0.08^\circ C \, d^{-1}$ whereas the Deardorff scheme predicts a maximum cooling rate of about $1.72^\circ C \, d^{-1}$ (Fig. 11c). In this case, the bulk Richardson number estimates exceeded criticality ($R_b > 1$) in the experimental domain and the PRT scheme predicted no entrainment heat flux. In contrast to Wake 1, entrainment predicted by Deardorff scheme does not correspond to SST decreases in the mixed layer (Fig. 2d), and radiative fluxes may become more important during this stage that would lead to restratification in the mixed layer.
b. Mass budget

1) Forced stage—Storm

Using the MLD fields and the geostrophic and near-inertial velocities, each term in the mass conservation expression (2) is estimated. Relative magnitudes and spatial distribution of each of the terms in (2) are shown in Fig. 12. In the eddy region (\(>4R_{\text{max}}\)), geostrophic advection term \((\mathbf{U}_g \cdot \nabla h)\) has a large magnitude of \(-0.02 \text{ cm s}^{-1}\) (Fig. 12a). Strong MLD gradients are also significantly advected by near-inertial currents (Fig. 12b) with a magnitude of \(-0.01 \text{ cm s}^{-1}\). By contrast, vertical advection (upward and downward motion of the isopycnals in the thermocline) correspond to the divergent and convergent cycle of the near-inertial currents with a magnitude of \(0.8 \text{ cm s}^{-1}\) close to the storm track (Fig. 12c).

Entrainment velocities estimated from the three mixed layer parameterizations have the same order of magnitude and the maximum entrainment velocity is located in the rear-half of the storm. The shear-induced entrainment scheme (Fig. 12d) predicts localized maximum values that spatially vary due in part to the nonlinear dependence on \(R_s\), whereas the stress-induced entrainment follows the wind stress pattern (Fig. 12e). The Deardorff scheme indicates a \(R_s\) dependence in the less than critical \(R_s\) regions although the nonlinear dependence is not pronounced as in the shear-induced case. Yet, in regimes where \(R_s > 1\), the \(R_s\) and \(R_w\) dependence predict entrainment rates similar to those from the stress-induced scheme. While these three schemes predict qualitatively similar entrainment rates, significant differences exist over the experimental domain. In addition, due to broad wind field and probe failures, entrainment at the primary \(R_{\text{max}}\) due to shear instabilities remains unclear. To improve entrainment estimates, high-resolution measurements of velocity and temperature in space and time are required. However, the relative magnitude of the terms indicate that the large variability in the mass budget is due to a combination of entrainment and the upwelling and downwelling cycles.

2) Relaxation stage—Wake 1

Near-inertial velocities during Wake 1 diverged from the track on the left side of the domain and converged toward the track on the right (Fig. 6b) and the maximum geostrophic velocities in the ML were about \(1 \text{ m s}^{-1}\) (Fig. 5b). Here, the maximum geostrophic advection magnitude of \(0.05 \text{ cm s}^{-1}\) is evident in the LCWCE regime due to the strong current response (Fig. 13a). Consistent with the near-inertial current divergence from the track after \(1.25 \text{ IP}\) following passage, the near-inertial advective terms of the mixed layer depth gradients \((\mathbf{U}_l \cdot \nabla h)\) have maximum values of \(-0.017 \text{ cm s}^{-1}\) and \(0.008 \text{ cm s}^{-1}\) (Fig. 13b). This effect is primarily due to the strong MLD gradients and the energetic near-inertial current response between the eddy and the storm track regions. However, as in the storm experiment, the maximum value of vertical advection (Fig. 13c) remains one to two orders of magnitude larger than the horizontal advective terms, and entrainment velocity estimates using the three schemes indicate fairly significant differences.

As the boundary layer winds from the ECMWF model used in this study were about \(7 \text{ m s}^{-1}\) (small \(u_w\)) compared to the Storm experiment, the maximum entrainment velocity is about \(0.025 \text{ cm s}^{-1}\) using the KT scheme, whereas \(w_e\) estimated using the PRT and Deardorff schemes are more than \(1 \text{ cm s}^{-1}\) close to the storm track. This significant difference is due to the low bulk Richardson numbers that result from the vigorous near-inertial current shears across the oceanic mixed layer base, suggesting continued entrainment mixing events. Thus, in the mass budget, horizontal advective terms are relatively negligible compared to vertical mixing.

3) Relaxation stage—Wake 2

During Wake 2 (Fig. 14), the geostrophic advection magnitudes are \(0.01 \text{ cm s}^{-1}\) away from the track and the near-inertial advection (Fig. 14b) at \(2R_{\text{max}}\) has values of \(\geq 0.012 \text{ cm s}^{-1}\). Vertical advection at the mixed layer base also has a fairly large magnitude of \(0.3 \text{ cm s}^{-1}\) (Fig. 14c). As during Wake 1, weak winds result in a very small \((<0.01 \text{ cm s}^{-1})\) entrainment velocities from the KT scheme. The Deardorff scheme predicts a maximum entrainment velocity of \(0.125 \text{ cm s}^{-1}\) in the regimes where the Richardson number is close to criticality between 0 to 4 \(R_{\text{max}}\). S98 found that by the Wake 2 experiment (three days after the storm passage), near-inertial shears have decreased due to the relatively rapid vertical energy propagation into the thermocline. Therefore, vertical advection associated with upwelling and downwelling processes significantly influenced the ML mass budget during wake 2.

c. Discussion

The relative magnitude of rms heat budget quantities at 0, 2, 4, \(R_{\text{max}}\), and in the eddy are summarized in Fig. 15. During storm (Fig. 15a), the geostrophic advection is negligible near the track, increases beyond \(2R_{\text{max}}\) and is larger than the surface fluxes at \(4R_{\text{max}}\) and in the eddy region. Prestorm ocean variability affected the upper ocean budget during Gilbert and its relative magnitude continued to increase after storm passage (Figs. 15b,c). Close to the storm track \((<2 R_{\text{max}})\) near-inertial advection has larger values than geostrophic advection with increasing relative magnitude in the mixed layer budget from Storm to Wake 2 snapshots. Entrainment heat flux remains the dominant mechanism in the mixed layer heat budget throughout the experiment. The KT scheme predicts maximum entrainment heat flux at 0 and 4 \(R_{\text{max}}\) mainly due to the primary and secondary wind maxima whereas PRT and Deardorff schemes have the maximum
Fig. 12. Mass budget [terms of (Eq. (2))] during the storm experiment in cm s$^{-1}$: (a) geostrophic advection, (b) near-inertial advection, (c) vertical advection, (d) entrainment velocity (PRT), (e) entrainment velocity (KT), and (f) entrainment velocity (Deardorff). Note that the middle color scale is for (c) only. The lower color scale is for (d), (e), and (f).
FIG. 13. Same as Fig. 12 during the Wake 1 experiment: (a) geostrophic advection, (b) Near-inertial advection, (c) vertical advection, (d) entrainment velocity (PRT), (e) entrainment velocity (KT), and (f) entrainment velocity (Deardorff). Note that the middle color scale is for (c) only. The lower color scale is for (d), (e), and (f).
at 2 and 4 $R_{\text{max}}$. In the eddy region, KT and Deardorff schemes predict relatively larger entrainment compared to PRT scheme during Storm. This suggests a strong dependence of the heat budget on the entrainment scheme. Further, during Wake 1, the PRT and Deardorff schemes predict continued entrainment (Fig. 15b) because of the low bulk Richardson numbers. While the prestorm oceanic conditions cannot be ignored in estimating the heat budget, entrainment and vertical advection of the mixed layer base dominated the mass budget as indicated by their relative magnitudes (not shown).

4. Summary and conclusions

Mixed layer budgets of heat and mass during and subsequent to the passage of Hurricane Gilbert is investigated based on conservation arguments using objectively analyzed fields of profiler data. The presence of a LCWCE and the associated geostrophic velocities modulate the mixed layer heat and mass budgets primarily because of the strong gradients between the storm track and the LCWCE during Storm, Wake 1, and Wake 2. This result, along with the temperature profiles in the eddy region leads to the conclusion that the oceanic background conditions may not be neglected for realistic simulations in oceanic and coupled response models. Near-inertial advection magnitudes increase from Storm to Wake 2 due to the dispersion of the storm-induced velocities away from the track, which is dominated by the near-inertial response (S98).

Flight-level winds reduced to 10 m are combined with ECMWF model and buoy winds to estimate the surface fluxes during Storm experiments. Given the large wind speeds and assumed constant air–sea humidity differences, the surface flux pattern follows the wind speed.
as per the bulk aerodynamic formulae and is dominated by the latent heat flux. Entrainment mixing is the dominant mechanism in the mixed layer and the overall budget has a strong dependence on the entrainment scheme adopted. During the Storm snapshot, patterns of entrainment estimated using the three schemes are similar, yet the maximum values in the same half of the forced region are displaced from each other. Depending upon the entrainment scheme used, the spatially averaged surface fluxes contributed from 6% (Deardorff) to 16% (PRT) to the total mixed layer heat budget. It should be noted that, the surface fluxes and entrainment based on KT scheme are instantaneous for each snapshot of $u_*$ and $Q_0$, whereas the PRT and Deardorff schemes include $\delta V$ at mixed layer base that is primarily due to near-inertial velocities that represent an integrated response to the wind stress forcing. This is apparent during Wake 1 as KT estimates of entrainment are negligible whereas PRT and Deardorff schemes predict continuing entrainment due to the large residual current shears and the resulting below critical bulk Richardson numbers. During Wake 2, localized entrainment is still predicted by the Deardorff scheme because of the combined effect of $Q_0$, $\delta V$, and $u_*$, whereas PRT scheme predicts no entrainment as the $R_b$ values are above criticality due to the reduced near-inertial shears. Hence, it is highly desirable to include the prestorm oceanic variability and a mixing scheme that has a $Q_0$, $\delta V$ and $u_*$ dependence in ocean response and coupled studies to improve upper ocean heat budget estimates and hurricane intensity pre-
Radiative fluxes are not included in these computations and diurnal variabilities probably are more relevant for Wake 2.

Aircraft and buoy technology has now emerged where the air–sea interactions during these extreme events can be quantified with movable observing strategies that complements NOAA Hurricane Research Division research missions. The new Global Positioning System sondes provide high quality measurements of the humidity, temperature, and winds in the atmospheric boundary layer structure (Hock and Franklin 1999) that must be complemented by concurrent oceanic profiles of the oceanic temperature, density, and currents from airborne ocean profiling devices (S98). More insights will be gained by utilizing satellite remote sensing from the combination of TOPEX sea surface heights and AVHRR-derived sea surface temperatures as in Hurricane Opal (Shay et al. 2000). These observations provide the larger-scale context to examine the role of mesoscale oceanic variability in the oceanic and coupled responses to the atmospheric forcing structures as well as the background conditions for the model initialization. Profiles in both geophysical fluids will allow the surface and entrainment fluxes to be quantified with confidence and numerical models to be tested to exploit deficiencies in parameterizations and to isolate physical processes involved in the air–sea interactions (Hong et al. 2000). This approach will provide the data to understand the ocean’s role in intensity changes of hurricanes (Marks et al. 1998).

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