Abstract—The spatial structure of surface currents collected using a Doppler Radar system off the Florida Keys has been investigated. Surface current vectors were acquired within a rectilinear grid containing 700 nodes, where each node was spaced 1 km apart. Principal component analyses indicate that at least 63% of the variance of the surface currents at the 700 nodal locations can be accounted for by only three principal components. The principal modes of circulation during two separate experiments were extracted and found to be basically the same, although the first two modes were reversed between the two experiments. Coherence spectra were determined for nodal spacings between 1.76 km and 10.48 km, and the results of these analyses indicate that over most of the experimental area, approximately 60% of the variance is coherent over separations of 10.48 km. Application of a synoptic time-series model [1] indicates that accurate prediction of the mean currents over each of the 65-h subrecord lengths was the dominant factor in controlling model performance, and that on the order of 10% error could be expected in using the time-series model to predict the low-frequency fluctuations. This indicates that time-series modeling of surface currents may be feasible and useful in estimating the long-term mixing characteristics of contaminants transported in the surface layer.

Index Terms—Coherence, Doppler radar, mixing, principal components, spectral analysis, surface currents, time-series models.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>α_{ij}</td>
<td>Elements of eigenvector matrix.</td>
</tr>
<tr>
<td>α_t</td>
<td>Residual time series.</td>
</tr>
<tr>
<td>c_0</td>
<td>Speed of light.</td>
</tr>
<tr>
<td>F_c, F_a</td>
<td>Clockwise and anticlockwise spectra, respectively.</td>
</tr>
<tr>
<td>f_k(t_n)</td>
<td>Principal-component time series.</td>
</tr>
<tr>
<td>f_R</td>
<td>Frequency of radar signal.</td>
</tr>
<tr>
<td>M</td>
<td>Number of stations.</td>
</tr>
<tr>
<td>N</td>
<td>Number of measurements.</td>
</tr>
<tr>
<td>t_n</td>
<td>n\textsuperscript{th} time step.</td>
</tr>
<tr>
<td>u_i</td>
<td>Component of surface velocity at station (i).</td>
</tr>
<tr>
<td>V_i(t_n)</td>
<td>Surface velocity vector at station (i).</td>
</tr>
</tbody>
</table>

SURFACE currents influence such processes as the transport and dispersion of oil spills and the mixing of domestic sewage discharged from outfalls in the coastal ocean. In fact, surface current measurements have been used previously in several engineering applications, notably in the design of ocean outfalls [2] and in the design of tidal power projects [3]. Despite their relative importance, surface currents have not been measured and characterized extensively in the coastal ocean, primarily due to a limitation in the technology needed to resolve their spatial features at scales that are relevant to transport predictions. Until recently, resolution of the horizontal features of surface currents depended on extrapolation from current measurements acquired from subsurface current meters. However, because of the expense associated with deploying large arrays of current meters, the spatial resolution of these measurements are seldom adequate for transport predictions on the 1-km scale, which is typical of oil spills and outfall mixing zones.

An emerging and cost-effective technology for measuring surface currents in coastal regions is the ocean surface current radar (OSCR) system, which is based on the Doppler radar technique [4]. Using this radar system, shore-based measurements of surface currents within a rectilinear grid containing up to 700 grid points at 1-km spacing can be acquired at 20-min intervals. These measurements afford a degree of spatial resolution that cannot be obtained from moored current meters.

The OSCR system has been used to examine the spatial structure of the surface currents in an offshore region adjacent to the Florida Keys, between Key West and Big Pine Key. The analyses reported in this paper are aimed at identifying the principal circulation modes, describing the quantitative parameters that determine the surface-current structure, assessing the stationarity of these structural parameters, and determining whether synoptic time-series models could be effectively used in the study region.

II. OCEAN SURFACE CURRENT RADAR

The OSCR system was originally developed at the Rutherford-Appleton Laboratory of the United Kingdom Science and Engineering Research Council [5]. The OSCR system is a...
dual-frequency Doppler radar system, which operates in the HF and VHF radio bands and remotely senses surface currents in the coastal ocean using Bragg backscatter and Doppler shift [6]. This radar system operates on the principle that the transmitted pulses of electromagnetic waves are reflected by surface waves whose resonant wavelength is one-half the radar wavelength (Bragg resonance), and the resulting peaks in the power spectrum of the reflected beam are shifted due to the underlying surface current, causing the surface waves to advance or recede from the radar sites (Doppler shift). The magnitude of this Doppler shift in frequency is predicted based on both the wave celerity and the component of the surface current in the direction of the radar beam. A typical 27-MHz signal will result in a wavelength, \( \lambda_R \), of about 11 m, and the (Bragg) resonant wavelengths of surface waves, \( \lambda_{wp} \), are equal to \( \lambda_R/2 \). Since these waves are typically moving both towards and away from the radar beam, the reflected beam has two resonant spectral peaks that are shifted to both the high- and low-frequency ends by an amount \( c/\lambda_{wp} \), where \( c \) is the wave celerity of wavelength \( \lambda_{wp} \). If a surface current exists, with component \( V_y \) along the radar beam, then there is an additional Doppler shift equal to \( V_y/\lambda_{wp} \) that occurs in the spectrum of the reflected signal. The component of the surface current along the radar beam is then determined by an examination of the peaks in the spectrum of the reflected radar signal, where the only unknown in the predicted Doppler shift is the beam component of the surface current, \( V_y \). Resolution of the magnitude and direction of surface currents requires that two components of the surface current be measured, and therefore two radar sites must be used. These two sites are called “master” and “slave” sites, where the reflected signals at the slave sites are transmitted to the master site for processing. The accuracy of the surface current vector is related to the angle between the beams originating from the master and slave units [6], [7], and small angles may cause unacceptable amplification of measurement errors of the individual measurements. It is recommended that the angle between radar beams be greater than 30° in order to maintain an acceptable accuracy [6], [7], and the most accurate measurements are made when the angle between radar beams equals 90° [8]. In using surface current measurements from OSCR systems, it is important to remember that surface current measurements are not point values, but area averages. The amount of a real averaging, over the so-called “footprint,” is determined by pulse length and beam width of the radar [6]. Averaging also occurs over a water-column depth estimated to be on the order of 0.47 m (\( \lambda_R/8\pi \)) at 27 MHz [9]. The high-frequency (HF) mode of the OSCR system used in this study operates at 25.4 MHz and measures surface currents at up to 700 grid points at a horizontal resolution of 1 km at 20-min intervals over a spatial domain of up to 30 km \( \times \) 45 km. The very-high-frequency (VHF) mode operates at 49.945 MHz and covers an approximate 12 km \( \times \) 12 km domain at a horizontal resolution of 250 m. The accuracy of the surface current measurements is on the order of 4 cm/s for vector current speed with a directional accuracy of about 5°. A more detailed discussion of the operating principles of the OSCR system may be found in [7] and [6].

The OSCR system is fairly new to the United States and was first deployed in 1991 in the High Resolution Remote Sensing Experiment sponsored by the Office of Naval Research [8] and then again in 1993 [10], [11]. The spatial resolution of the surface currents are superior to the resolution that is feasible using current meter measurements. In fact, the amount of spatial detail available from OSCR measurements is equivalent to that generated by the deployment of 700 current meters. With such resolution, approximations with respect to defining the velocity field as a uniform large-scale velocity plus empirical scale-dependent small-scale velocity fluctuations are obviated.

III. Previous Studies

Shay et al. [11] have investigated the correlation between OSCR-measured surface currents and subsurface current meter measurements at 13.8 and 9.5 m beneath the surface at locations approximately 20 and 34 km offshore from Cape Hatteras. Using these measurements, Shay et al. [11] decomposed the measured currents into various frequency ranges to examine the vertical current structure with respect to vertical correlation scales and rms differences. The rms differences ranged from 12 to 14 cm/s due to vertical current shears \( O(10^{-2} \text{ s}^{-1}) \) associated with the Gulf Stream’s baroclinic current structure.

Validation studies of the OSCR system were reported by Prandle [7], who compared the tidal (\( M_2 \)) components of the measured currents with those at moored current meters. The level of agreement was considered to be within the range of variability expected in the vertical current structure and within the averaging area of the OSCR measurements. Prandle [7] also verified that the wind-driven currents in the OSCR measurements were within the range of 1%–3% of the wind speed, which is commonly assumed in engineering analyses. Collar and Howarth [12] compared currents measured by OSCR with those observed from moored surface instruments and by satellite-tracked surface drifters. The results of these experiments confirmed that the agreement between these measurements was good, and Collar and Howarth further concluded that the OSCR measurements were characteristic of the top 1 m of the water column or less.

IV. Field Experiments

A. Experimental Setup

For the OSCR deployments in the Keys in support of the U. S. Coast Guard-sponsored Ocean Pollution Research Center Program, the master site was located on Boca Chica Key Road along the main beach access road [13]. The site, located at 24° 34′ N and 81° 40.4′ W, directly overlooked the Atlantic Ocean side of the Florida Keys and was just east of the U.S. Naval Air Station. A total of sixteen receive antenna elements was oriented on a line with an angle of about 222.5° true north over a 90-m aperture. The corresponding boresight angle was 132.5° true north. The slave station was deployed in Bahia Honda State Park, directly overlooking the Atlantic Ocean side of the Key. This site (24° 39.3′ N, 81° 16.9′ W) consisted of 14
elements, because of space limitations and the requirement to obtain an optimal boresite angle of 204.5° true. The position of the center of the receiving elements was 294.5° true north. Note that the baseline distance between the master and slave was about 34 km. While this separation was slightly larger than the normal operating envelope for OSCR (24–30 km), this configuration of the array sites was the best that could be achieved given the inaccessible beach areas between the two sites. These locations of the OSCR master and slave sites were determined by positions from the global positioning system (GPS), and communications between the two sites were facilitated by a UHF antennae (458 MHz).

The data used in this study were collected from two separate OSCR deployments. The first OSCR deployment was from 30 September, 1993 (1640 h) to 14 October, 1993 (1520 h), and the second deployment was from 18 May, 1994 (2320 h) to 13 June, 1994 (1140 h). Each of these deployments measured surface currents at the 700 grid points shown in Fig. 1, and measurements of surface currents were acquired every 20 min. This resulted in a total of 1003 measurements during the first deployment and 1845 measurements during the second deployment, and these data sets are referred to as DS 1 and DS 2, respectively.

B. OSCR Performance and Data Quality

As noted in [13], the return of the OSCR data generally exceeded 90% over most of the domain. However, dropouts frequently occurred near the master site where the data return was about 35% compared to other regions of the OSCR domain. One possibility was the interference of Citizens Band radios operating between 20–30 MHz, which was also found during the OSCR deployments along Cape Hatteras [10]. The data time series indicated that during each day, anomalously large spikes occurred in the current time series, which suggested that this interference pattern was real. However, no conclusive evidence was found linking these large values to aircraft traffic operations associated with the Naval Air Station.

The raw current time series were linearly interpolated over the drop outs, smoothed by a 3-point Hanning window [14], and low-pass filtered at the Nyquist frequency (1.5 cph, corresponding to a measurement interval of 20 min) to remove noise from the 700 grid points of surface currents. To remove
the large single-point anomalies, the second derivative of the velocity was used to flag these values, which were replaced by linearly interpolated values. This approach appeared to be the most useful in isolating and removing these large values while preserving the veracity of the observations. These time series were then designated as the processed data files.

V. PRINCIPAL COMPONENT ANALYSIS

A. Theory

The analysis of multiple time series of ocean currents by the method of principal components, also called empirical orthogonal function analysis, has been used by many investigators as a means of extracting the important modes of circulation from synoptic current meter measurements. Notable studies can be found in [15]–[17]. Principal component analysis separates $n$ scalar time series into a linear combination $n$ orthogonal time series (principal components), where the variance in each principal component time series is maximized. The value of this approach in the analysis of synoptic time series is that the correlations of the principal components with the measured time series can be used to identify the scale and extent of the independent (orthogonal) phenomena extracted by principal component analysis. An in-depth discussion of the details of the principal component method can be found in [18], and this paper only discusses the salient features of this approach and the extension to vector time series.

Consider a measured $p$-dimensional time series $X(t)$ where

$$X(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_n(t) \end{bmatrix}$$  

(1)

then a derived $p$-dimensional time series $Y(t)$ given by

$$Y(t) = \begin{bmatrix} Y_1(t) \\ Y_2(t) \\ \vdots \\ Y_n(t) \end{bmatrix}$$  

(2)

can be obtained by the linear transformation of $X(t)$ by the matrix $A$, where

$$Y = AX.$$  

(3)

Maximizing the variance of the component $Y_i$, subject to the constraint that the time series $Y_i$ and $Y_j$ are orthogonal for $i \neq j$, leads to the requirement that the rows of $A$ must be equal to the $p$ eigenvectors of the covariance matrix of $X$ [18]. The resulting independent (orthogonal) time series $Y_i$ are called the principal components of the $p$-dimensional time series $X(t)$. On the basis of (3), the measured $p$-dimensional times series can be written in terms of the $p$ principal components as

$$X = BY$$  

(4)

where

$$B = A^{-1}.$$  

(5)

Of course, by using all of the principal components, (4) exactly reconstitutes the measured time series from the principal components. The utility of extracting principal components from measured time series is realized when the measured time series is dominated, in terms of explained variance, by relatively few principal components. Under these circumstances, the most important phenomena affecting the $p$ measured time series can be isolated and studied in detail.

In the case of surface currents, two scalar time series are measured at each node location, and therefore for $N$ node locations there are $2N$ measured scalar time series. Denoting the scalar measurements at the $j$th node by the velocity components $U_j$ and $V_j$, then the $2N$-dimensional measured time series $X$, in (1) can be written as

$$X(t) = \begin{bmatrix} U_1(t) \\ V_1(t) \\ U_2(t) \\ V_2(t) \\ \vdots \\ U_N(t) \\ V_N(t) \end{bmatrix}.$$  

(6)

Following (4), the velocity components at the $j$th node that are associated with the $j$th principal component are given by

$$X_{2i} = B_{2i,j}Y_j$$  

(7)

and

$$X_{2i+1} = B_{2i+1,j}Y_j$$  

(8)

where $B_{a,b}$ is the element of the matrix $B$ in row $a$ and column $b$, derived from the $2N$-dimensional covariance matrix of the measured $2N$-dimensional time series in accordance with the principal component method described earlier. On the basis of (7) and (8), the velocity field associated with the $j$th principal mode of circulation can be easily visualized by plotting the vectors with (constant) components $B_{2i,j}$ and $B_{2i+1,j}$ at each of the $N$ measurement locations. These vectors show the relative magnitudes and directions of the velocities at all measurement locations associated with the $j$th principal mode of circulation.

B. Results

Principal component analysis results in the extraction of 1400 orthogonal time series from each of the two OSCR deployments, and the percentage of the total variance accounted for by the first 10 components in each data set is shown in Table I. In each data set, more than 63% of the variance is accounted for by the first three principal components, with a majority of the variance accounted for by the first two principal components. These results indicate that there are two predominant modes of circulation, and these modes are accounted for by the first two principal components. The first three principal components of DS 1 and DS 2 are shown in Figs. 2 and 3, respectively. In each of these principal component time series, short-term cycles and long-term trends are evident. Other principal modes of circulation than the three modes presented here may also be of importance but
TABLE I  
DISTRIBUTION OF VARIANCE IN PRINCIPAL COMPONENTS

<table>
<thead>
<tr>
<th>Principal Component</th>
<th>Percent of Variance DS 1</th>
<th>Percent of Variance DS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32.8</td>
<td>35.7</td>
</tr>
<tr>
<td>2</td>
<td>26.3</td>
<td>19.6</td>
</tr>
<tr>
<td>3</td>
<td>5.2</td>
<td>7.7</td>
</tr>
<tr>
<td>4</td>
<td>3.2</td>
<td>3.9</td>
</tr>
<tr>
<td>5</td>
<td>2.7</td>
<td>3.7</td>
</tr>
<tr>
<td>6</td>
<td>2.1</td>
<td>2.6</td>
</tr>
<tr>
<td>7</td>
<td>1.9</td>
<td>2.3</td>
</tr>
<tr>
<td>8</td>
<td>1.4</td>
<td>1.8</td>
</tr>
<tr>
<td>9</td>
<td>1.3</td>
<td>1.5</td>
</tr>
<tr>
<td>10</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>Total</td>
<td>78.1</td>
<td>79.8</td>
</tr>
</tbody>
</table>

are probably more localized in nature. It is interesting to note that in both data sets, 39 principal components account for 90% of the variance. The spectrum of the principal components were calculated and segregated into three ranges: low frequency, tidal, and high frequency. Low-frequency fluctuations are defined as having periods greater than 26 h, tidal fluctuations have periods between 10 h and 26 h, and high-frequency fluctuations have periods less than 10 h. The spectral characteristics of the first three principal components in both data sets are given in Table II. From the data shown in Table II, all three of the principal modes of circulation are predominantly low frequency. The first principal component in DS 1 and DS 2 contains 93% and 82% of the variance, respectively, in the low-frequency range, while the second and third principal components contain between 53% and 72% of the variance in the low-frequency range. The second and third principal components are similar to each other in their distribution of variance between the tidal and low-frequency modes, with approximately two-thirds of the variance in the low-frequency range and one-third of the variance being in the tidal range. For all three principal components, the variance associated with high-frequency fluctuations are minimal. On reviewing all of the autocorrelation functions of the principal components, the fluctuations are almost perfectly correlated at 20-min lags, equal to the measurement interval, indicating that differencing adjacent values in the principal component time series may be effective in reducing the variance of the principal component time series. The characteristics of the differenced residual time series are shown in Table III. These results show that the residual variance after differencing is generally less than 5% of the original variance and predominantly shifting to the high-frequency mode. These results suggest that the currents tend to fluctuate about their value at the previous time step. Analytically, this model has the form

\[ X_t = X_{t-1} + \alpha_t \]  

(9)

where \( X_t \) represents the principal component time series and \( \alpha_t \) is the residual time series. It should be noted, however, that although most of the variance in the residual time series, \( \alpha_t \), is in the high-frequency range, spectral peaks are highest at the predominant tidal frequencies (i.e., \( M_2, K_1 \)).

The circulation modes associated with each of the first three principal components are shown in Figs. 4 and 5 for DS 1 and DS 2, respectively. Recalling that the principal velocity at each grid point is derived by multiplying the scalar principal component time series by the appropriate principal vector, then the circulation modes shown in Figs. 4 and 5 accurately reflect the relative values of the principal component vectors as well as the circulation directions if the scalar principal component time series are positive. Since the principal component time series are predominantly positive for the first two principal components and negative for the third principal component, the derived principal circulation modes for the first two components depict the circulation directions, while the derived principal vector for the third...
circulation mode is reversed to more adequately reflect the third principal circulation mode. Considering the circulation modes associated with DS 1, shown in Fig. 4, it is clear that the first principal component incorporates the influence of the Florida current on the measured surface currents; the second principal component is (probably) caused by a long-shore jet associated with the nearshore bathymetry constraining the wind-driven currents; and the third principal component is associated with a cyclonic convergence feature, which may be due to submesoscale eddy-like motions between the first two principal motions. Considering the principal circulation modes in DS 2, shown in Fig. 5, it is apparent that the principal modes are very similar to those found in DS 1, with one very important difference: the first two principal modes are reversed, meaning that the first principal component in DS 2 is similar to the second principal component in DS 1, and the second principal component in DS 2 is similar to the first principal component in DS 1. Recall that in DS 1 the Florida current effect was associated with the first (largest) principal component, and in DS 2 the effect of the Florida current was associated with the second principal component and is second in importance to the nearshore jet circulation mode. This enlightening result indicates that the Florida current does not always dominate the circulation in the study region, a condition that is indicative of nonstationarities in the flow regime that have longer time scales than the duration of these experiments and larger spatial scales than the OSCR domain. The third principal component again indicates the presence of a flow regime that is opposite in direction to that of the first two principal components and is more characteristic of eddy formation in the near-shore region.

VI. Diagnostic Model

The principal component analysis described in the previous section indicates that there is a consistent structure in the velocity field which can be identified even with relatively short deployments of the OSCR system. Accordingly, it is argued that the OSCR system may be used as a basis for mapping the spatial structure of the surface currents in a particular region, and these currents may be subsequently modeled using a synoptic time-series model. Conceptually, then, the use of the structural maps extracted from OSCR data can be used to assess surface mixing characteristics over a much wider range of conditions than encountered during OSCR deployments. This application could potentially be very useful in such applications as mixing-zone delineation surrounding ocean outfalls and estimating dispersion parameters in oceanic contaminant transport models. Given the large

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**TABLE II**

<table>
<thead>
<tr>
<th>Set</th>
<th>Principal Component</th>
<th>Mean (cm/s)</th>
<th>Variance (cm/s)²</th>
<th>Spectral Distribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1799</td>
<td>403225</td>
<td>93  6  1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>521</td>
<td>324902</td>
<td>63  33  4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-455</td>
<td>64009</td>
<td>64  30  6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1401</td>
<td>608400</td>
<td>82  12  6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1133</td>
<td>335241</td>
<td>53  39  8</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-244</td>
<td>131769</td>
<td>72  22  6</td>
</tr>
</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>Set</th>
<th>Principal Component</th>
<th>Mean (cm/s)</th>
<th>Variance (cm/s)²</th>
<th>Spectral Distribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>-0.2</td>
<td>3721</td>
<td>2  11  87</td>
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<td>2</td>
<td>2</td>
<td>-0.6</td>
<td>8649</td>
<td>2  12  86</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.1</td>
<td>2116</td>
<td>2  12  86</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.6</td>
<td>9216</td>
<td>2  14  84</td>
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<tr>
<td>2</td>
<td>2</td>
<td>0.2</td>
<td>8649</td>
<td>3  22  75</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-0.1</td>
<td>2500</td>
<td>2  20  78</td>
</tr>
</tbody>
</table>
A. Theory

A model for predicting a synoptic vector time series conditioned on measurements at another location has been developed by Chin and Roberts [1]. This model is well suited for applications in which OSCR measurements are used to identify structure, and then this structure is used along with a centrally measured time-series estimate velocities at other locations under a variety of conditions. The model expands the velocity time series at each node in terms of its Fourier coefficients as

$$u(t_n) + \dot{v}(t_n) = \sum_{j=0}^{(N-1)/2} \left[ F_+(\omega_j) \exp(i\omega_j n) + F_-(-\omega_j) \exp(-i\omega_j n) \right]$$

where $u(t_n)$ and $v(t_n)$ are the components of the complex velocity, $F_+(\omega_j)$ and $F_-(-\omega_j)$ are the clockwise and anticlockwise components, respectively, of the complex Fourier transform of the velocity at the frequency $\omega_j$, where

$$\omega_j = \frac{2\pi j}{N}$$
CHIN et al.: THE STRUCTURE OF OCEAN-SURFACE CURRENTS MEASURED BY DOPPLER RADAR 163

The time-series model proposed by Chin and Roberts [1] is designed to predict synoptic velocity fluctuations at a remote station, B, given velocity measurements at station A. The model uses contemporaneous measurements at stations A and B to calibrate the model and is based on four key assumptions. First, the model assumes that, in each subrecord, the mean flow at station B can be derived by a rotation and scaling of the mean velocity at station A, in which case

\[ V_B = \xi \langle V_A \rangle \]  

(12)

where \( V_B \) is the predicted complex mean velocity at station B, \( \langle V_A \rangle \) is the measured complex mean velocity at station A, and \( \xi \) is a complex constant derived from synoptic measurements using the relation

\[ \xi = \frac{\sum_{m=1}^{N_r} (V_{A+m} - \bar{V}_B)^m}{\sum_{m=1}^{N_r} |V_{A+m}|^2} \]  

(13)

where \( N_r \) is the number of measurement subrecords, \( V_A \) and \( \bar{V}_B \) are the complex mean velocities in subrecord \( m \), and the asterisk denotes the complex conjugate. The second key assumption in the time-series model is that, in each subrecord, the magnitude of the spectral components at B, \( |\hat{F}_{B+}(\omega_j)| \) and \( |\hat{F}_{B-}(\omega_j)| \), are linearly dependent on the magnitudes of the spectral components at A, \( |\hat{F}_{A+}(\omega_j)| \) and \( |\hat{F}_{A-}(\omega_j)| \), according to the relations

\[ |\hat{F}_{B+}(\omega_j)| = \alpha_{+}(\omega_j)|\hat{F}_{A+}(\omega_j)| + B_{+}(\omega_j) \]  

(14)

and

\[ |\hat{F}_{B-}(\omega_j)| = \alpha_{-}(\omega_j)|\hat{F}_{A-}(\omega_j)| + B_{-}(\omega_j) \]  

(15)

where \( \alpha_{+}(\omega_j), \alpha_{-}(\omega_j), \beta_{+}(\omega_j), \) and \( \beta_{-}(\omega_j) \) are scalar least-squares regression constants derived from the measured subrecords. The third key assumption in the time-series model is that the time lag between the currents at A and B at frequency \( \omega_j \), \( \delta(\omega_j) \), remains stationary and is estimated by

\[ \delta(\omega_j) = \frac{\sum_{m=1}^{N_r} \text{arg} \left( F_{A+} - F_{A+}^* F_{B+} - F_{B+}^* \right)}{2N_r} \]  

(16)

where the spectral components, \( F_{A+}, F_{A+}^*, F_{B+}, \) and \( F_{B+}^* \) are calculated for each of the \( N_r \) subrecords in the calibration data. The final key assumption in the time-series model is that the orientation of the principal axis at the remote station, \( O_B(\omega_j) \), is stationary for each frequency, \( \omega_j \), and is estimated by

\[ O_B(\omega_j) = \text{arg} \left\{ \sum_{m=1}^{N_r} F_{B+} \exp(i\theta^*) + F_{B-} \exp(-i\theta^*) \right\}_m \]  

(17)

where the spectral components \( F_{B+} \) and \( F_{B-} \) are calculated for each of the \( N_r \) subrecords, and \( \theta^* \) in each subrecord is given by

\[ \theta^* = \frac{1}{2} \text{arg} (F_{B+} F_{B+}^*) \]  

(18)

and the asterisk indicates the complex conjugate. The parameters of the synoptic time-series model developed by Chin and Roberts are \( \alpha_{+}(\omega_j), \alpha_{-}(\omega_j), \beta_{+}(\omega_j), \beta_{-}(\omega_j), \delta(\omega_j), \) and \( O_B(\omega_j) \), and these model parameters are called structural parameters, since they embody the structural characteristics of the spatial variability in the velocity field. The time-series model is designed to predict the currents at station B, given measured currents at station A. If the contemporaneous measured spectra at station A used to predict subrecord at B are given by \( \hat{F}_{A+}(\omega_j) \) and \( \hat{F}_{A-}(\omega_j) \), then the spectral components at station B, \( \hat{F}_{B+}(\omega_j) \) and \( \hat{F}_{B-}(\omega_j) \), are estimated by [1]

\[ \hat{F}_{B+}(\omega_j) = [\alpha_{+}(\omega_j)|\hat{F}_{A+}(\omega_j)| + \beta_{+}(\omega_j)] \exp \{i\delta(\omega_j) - 0.5 \text{arg} (P_{A-} P_{A+}^*) + \delta(\omega_j) \} \]  

and

\[ \hat{F}_{B-}(\omega_j) = [\alpha_{-}(\omega_j)|\hat{F}_{A-}(\omega_j)| + \beta_{-}(\omega_j)] \exp \{i\delta(\omega_j) - 0.5 \text{arg} (P_{A-} P_{A+}^*) + \delta(\omega_j) \} \]  

(19)

These estimates of \( \hat{F}_{B+}(\omega_j) \) and \( \hat{F}_{B-}(\omega_j) \), along with the estimate of the mean velocity given by (12), are all derived from the calibrated model parameters and synoptic measurements at station A and are used in the time-series model to predict the complex (synoptic) velocity time series at station B. \( \hat{V}_B \), using the inverse Fourier transform

\[ \hat{V}_B = \hat{V}_B + \sum_{j=1}^{(N-1)/2} \hat{F}_{B+}(\omega_j) \exp(i\omega_0n) \]  

(20)

This model has been previously validated using spatially distributed current meter measurements off the coast of San Francisco [1], and the model successfully predicted synoptic velocities at several locations, where the structural parameters were estimated from previous observations.

B. Results

An important requirement in attempting to use a synoptic time-series model to describe surface currents is that the velocities at the nodal locations are coherent. To address the question of internodal correlation, the coherence spectra were computed for a variety of nodal spacings. Since all of the node locations were within a rectilinear grid and spaced 1 km apart, in the immediate proximity of each node, there are nodes 1 km away in the north, south, east, and west directions, and nodes located a distance of \( \sqrt{2} \) km away in the northeast, southeast, southwest, and northwest directions. At each node in the velocity grid, the coherence spectrum between the measured velocities at the central node and the velocities at each of the eight surrounding nodes were calculated, and from each of these coherence spectra, the frequencies with significant coherence were identified [19], and the average percentage variance in the central node that is significantly coherent was calculated. The results of these analyses yield the average coherent variance for stations spaced between 1 and \( \sqrt{2} \) km apart. These same analyses were repeated for
Fig. 6. Correlations with principal circulation modes in DS 1. Nodal distances are (a) 1 to $\sqrt{2}$ km, (b) 2 to $2\sqrt{2}$ km, and (c) 3 to $3\sqrt{2}$ km.

two other cases: nodal spacings between 2 and $2\sqrt{2}$ km; and nodal spacings between 3 and $3\sqrt{2}$ km. The results of these analyses are shown in Fig. 6 for DS 1 and in Fig. 7 for DS 2. Considering the results from DS 1, shown in Fig. 6, it is clear that for nodal spacings between 1 and $\sqrt{2}$ km, more than 80% of the variance is coherent over almost all of the experimental domain. However, for nodal spacings between 3 and $3\sqrt{2}$ km, the coherent variance is less than 80% everywhere, but more than 60% coherent variance over about half of the experimental area. The results from DS 2, shown in Fig. 7, show a much greater coherent variance at all nodal spacings than those found in DS 1. For nodal spacings between 1 and $\sqrt{2}$ km, almost the entire area has coherent variances greater than 80%, while for nodal separations between 3 and $3\sqrt{2}$ km, about one-third of the area has coherent variances greater than 80%, and more than one-half of the area has coherent variance in excess of 60%. Collectively, the spatial coherence analyses indicate that the velocity fluctuations with the longest length scales occur in a region furthest away from the shoreline, and, if significant spatial coherence is defined by a coherent variance of more than 60%, then most of the study region can be said to have significant spatial structure, even at spatial scales between 3 and $3\sqrt{2}$ km.

The time-series model developed by Chin and Roberts [1] was applied to several pairs of stations in the outer region of the study area where the spatial coherences were highest. Each
TABLE IV
MODEL PERFORMANCE CHARACTERISTICS

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Node A</th>
<th>Node B</th>
<th>Separation (km)</th>
<th>Coherent Variance (%)</th>
<th>Low Frequency (%)</th>
<th>Tidal Frequency (%)</th>
<th>High Frequency (%)</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
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<td>80.1</td>
<td>63.2</td>
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<td>0.02</td>
<td>0.05</td>
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</table>

The model performance parameters, $R_1$, $R_2$, and $R_3$, for each of the data sets and for each of the separations are shown in Table IV. The results indicate that the mean of the predicted time series is typically estimated with an error of 2% to 16% in the first data set and 3% to 19% in the second data set. These results, reflected in the $R_1$ statistics, indicate that the rotation and scaling relationships used to estimate the mean are working well, and the mean flows can be accurately estimated for synoptic time series up to 10 km apart. The $R_2$ statistic, defined by (23), indicates that the mean square of the error is between 5% and 22% of the mean square of the predicted time series in DS 1 and between 8% and 31% in DS 2. The lower prediction errors, 5% and 8% in DS 1 and DS 2, respectively, correspond to the closer stations where the velocity time series are more coherent. These results indicate that the magnitudes of the error fluctuations are relatively small compared with the magnitudes of the velocities in the time series and indicates that the model is performing well in this respect. The third model statistic, $R_3$, measures the variance of the errors relative to the variance of the predicted time series. The model performance in this respect is relatively poor, with the error variance being between 146% and 180% in DS 1 and between 64% and 139% in DS 2. It is clear that the model performance with respect to error variance in DS 2 is improved from the model performance in DS 1, presumably because more subrecords were used in model calibration in DS 2 than in DS 1. The results shown in Table IV collectively reflect the condition that the predicted time series contains fluctuations that are much less than the mean, which explains the good model performance relative to the mean square error ($R_2$) and the poor model performance relative to the error variance ($R_3$). These results indicate that the key measure of model performance is the accuracy with which the mean flow over a subrecord is estimated, and this statistical measure is contained in $R_1$. The conclusion that $R_1$ is the key statistic is further supported by the fact that the coefficient of variation of the magnitudes are on the order of 20% in DS 1 and 25% in DS 2, indicating that the magnitudes of the fluctuations...
tend to be much less than the mean in each subrecord. Based on the conclusion that \( R_1 \) is the mean, the statistic, we infer that the time-series model performs well in predicting the low-frequency components of the flow, characterized by variations in the 65-h (subrecord) means, while the model does not perform particularly well in predicting the higher frequency components. However, since most of the variance is within the low-frequency band, the overall performance of the time-series model is reasonably good.

VII. SUMMARY AND CONCLUSION

The study reported in this paper has investigated the spatial structure of synoptic measurements of surface currents collected using the OSCR system off the Florida Keys [13]. Surface-current vectors were acquired within a rectilinear grid containing 700 nodes, where each node was spaced 1 km apart. There were two data collection experiments, with the first experiment from 30 September to 14 October, 1993, and the second experiment from 8 May to 13 June, 1994. Surface-current velocities were subjected to principal component analysis and the primary (orthogonal) modes of circulation were extracted. In addition, a synthetic time-series model developed by Chin and Roberts [1] was applied to determine whether the structural characteristics of the velocity field were sufficiently stationary to allow modeling based on structural parameters derived from short-term contemporaneous measurements.

The principal component analyses [18] indicated that at least 63% of the variance of the surface currents at the 700 nodal locations could be accounted for by only three principal components. This result is indicative of the dominance of large-scale motions in the study region associated with the Florida current. The principal modes of circulation during both experiments were basically the same, although the first two modes were reversed between the first and second experiments. Spectral analysis of the primary circulation modes indicate that they are all predominantly low frequency, with periods longer than 26 h. An important characteristic of the temporal fluctuations in the surface velocities is that they tend to be predominantly Markovian, consisting of high-frequency fluctuations about their lag-1 magnitude. Such behavior is indicative of long-term trends.

Coherence spectra were determined for nodal spacings between 1.76 and 10.48 km from the OSCR-derived surface currents. The results of these analyses indicated that over most of the experimental area, approximately 60% of the variance is coherent over separations of 10.48 km. This indicates that the application of a synoptic time series model is worthy of investigation. Application of a synoptic time-series model developed by Chin and Roberts [1] indicates that accurate prediction of the mean currents over each of the 65-h subrecord lengths was the dominant factor in controlling model performance, and that on the order of 10% error could be expected in using the time-series model. This relatively good performance indicates that time-series modeling of surface currents may be feasible, based on the relatively short record lengths derived from the OSCR measurements. It should be noted that the model performance is relatively poor in predicting the higher frequency fluctuations in the velocities, probably due to their intermittency and small coherence scales [11].

In summary, the present investigation has extracted the predominant circulation modes within the study area and has demonstrated that the time-series model [1] may be appropriate for simulating the dominant (low-frequency) fluctuations in the velocity fields over separations of a few kilometers. These results collectively support the use of OSCR systems to map circulation structure, from which the parameters of a time-series model can be extracted. This approach could prove to be very useful in using structural parameters derived from OSCR measurements to estimate the long-term mixing characteristics of contaminants transported in the surface layer and to delineate mixing zones surrounding sewage outfalls in the ocean.

REFERENCES


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Dr. Graber served on several committees and panels for the National Academy of Sciences, the National Science Foundation, the Scientific Committee on Oceanic Research, and the World Meteorological Organisation.