On the Use of Two-Dimensional Incompressible Flow to Study Secondary Eyewall Formation in Tropical Cyclones

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(Manuscript received 28 July 2010, in final form 7 September 2010)

ABSTRACT

Previous studies have offered hypotheses for the mechanisms that lead to secondary eyewall formation in tropical cyclones by using two-dimensional incompressible flow. Those studies represented the convection-induced vorticity field as either large but weak vortices that are the same sign as the tropical cyclone core or as purely asymmetric vorticity perturbations that are an order of magnitude weaker than the core. However, both observations and full-physics simulations of tropical cyclones indicate that the convection-induced vorticity field should also include clusters of small vorticity dipoles whose magnitude is comparable to that of the high-vorticity core. Results of numerical simulations indicate that the interaction between the tropical cyclone core vortex and the convection-induced small vorticity dipoles of considerable strength in two-dimensional flow does not lead to coherent concentric vorticity ring formation. The axisymmetrization process under the simplification of two-dimensional incompressible flow appears to be incomplete for describing secondary eyewall formation.

1. Introduction

The exact mechanisms that lead to secondary eyewall formation in tropical cyclones remain unclear. The complete understanding of this process is critical in predicting the future evolution of tropical cyclones, as a period of significant intensity change typically occurs after secondary eyewall formation (e.g., Willoughby et al. 1982; Kossin and Sitkowski 2009; Kuo et al. 2009). Several hypotheses have been proposed (e.g., Willoughby et al. 1984; Nong and Emanuel 2003; Terwey and Montgomery 2008; Judt and Chen 2010), and a number of them use two-dimensional incompressible flow (Kuo et al. 2004, hereafter K04; Kuo et al. 2008, hereafter K08; Martinez et al. 2010, hereafter MBY10).

K04 proposed that the interaction between a small strong vortex and a large weak vortex could result in a concentric vorticity ring structure. K04 was an extension of the work by Dritschel and Waugh (1992, hereafter DW92) who examined the interaction of two circular vortex patches having equal uniform vorticity but unequal size. The initial configuration of the two vortices (see Fig. 1 of DW92) is controlled by two parameters: the ratio of the radii of the smaller vortex to the larger vortex \( R_2/R_1 \) and the dimensionless gap between the vortices \( D/R_1 \) where \( D = d - (R_1 + R_2) \) and \( d \) is the initial separation distance between the vortex centers. In the parameter space considered by DW92 (\( 0 < R_2/R_1 < 1 \) and \( 0 < D/R_1 < 1.6 \)), the stronger vortex (hereafter \( R_1 \)) is larger than the weaker vortex (hereafter \( R_2 \)). DW92 found five different regimes of the interaction: no merger, partial/complete merger, and partial/complete straining-out. In the complete straining-out (CSO) regime (see Fig. 3b of DW92), the stronger vortex completely shears apart the smaller weaker vortex and turns it into thin filaments that wrap around the stronger vortex but do not get incorporated into the stronger vortex.

Noting that the vorticity ring produced in the CSO regime by DW92 was “much too thin to be identified with that observed in the outer eyewall of a tropical cyclone,” K04 extended DW92 by examining the interaction with an additional parameter: the ratio of uniform vorticity in the weaker vortex to the stronger vortex \( \zeta_2/\zeta_1 \) (see Fig. 2 of K04). Now the problem is controlled by three parameters. In the parameter space considered by K04...
The stronger vortex is usually smaller and represents the rapidly rotating tropical cyclone core. The weaker vortex is larger and represents the vorticity field induced by moist convection outside the core. K04 found a set of parameters under which the core vortex completely shears apart the large weak vortex and turns it into thin filaments. These filaments wrap around the core vortex, become barotropically unstable, break down and mix to form a concentric ringlike structure (see section 3a for more details). K08 extended the results of K04 by representing the core vortex with a modified Rankine profile, which is more consistent with observations (e.g., Mallen et al. 2005), and reached similar conclusions. Both K04 and K08 represented the convection-induced vorticity field as a large vortex of weak vorticity that is the same sign as the core vortex. This characterization appears to be based on radar observations of tropical cyclones that had a large area of peripheral convection that eventually wrapped around the core to produce a secondary eyewall [e.g., Typhoon Lekima (2001) from Fig. 1 of K04]. This large area of convection appeared as a region of uniform reflectivity value, and K04 assumed that reflectivity could be used as a proxy for the convection-induced vorticity. However, although reflectivity offers useful information about the vigor of convection and the rainfall rate associated with it, it does not provide an accurate picture of the convection-induced vorticity field.

In a recent study by MBY10, it was proposed that secondary eyewall formation in tropical cyclones could result from the axisymmetrization of purely asymmetric wavenumber-4 ($n=4$) disturbances by the high-vorticity core of tropical cyclones. Unlike K04 and K08, MBY10 represented the hurricane core as a region of low vorticity surrounded by a ring of elevated vorticity, and a skirt of vorticity was added outside the core, consistent with observations (Mallen et al. 2005). The $n=4$ vorticity asymmetries used to represent the convection-induced vorticity field are broad (~40 km in width) and their peak values are approximately an order of magnitude smaller (~15%) than the vorticity in the hurricane core region (see Fig. 2 of MBY10). MBY10 chose $n=4$ asymmetries because they dominate the wave activity and represent the fastest growing modes for the particular axisymmetric vortex used in that study.

However, in contrast to the choices by K04, K08, and MBY10, observations (e.g., Fig. 9 of Powell 1990; Figs. 6 and 7 of Hence and Houze 2008; Fig. 15 of Didlake and Houze 2009) of tropical cyclones indicate that a better representation of the convection-induced vorticity field should also include positive–negative vorticity dipoles that have magnitudes comparable to that of the high-vorticity core vortex ($\zeta_2/\zeta_1 \approx 0.5$) and are smaller in size than the core vortex ($R_2/R_1 < 1$). Full-physics numerical simulations also support this notion. Shown in Fig. 1a is a horizontal cross section at $z = 4.40$ km of the vertical component of vorticity ($\zeta = \partial v/\partial x - \partial u/\partial y$) from a full-physics simulation of Hurricane Bill (2009) using the Weather Research Forecasting model version 3.1.1 (WRF; Skamarock et al. 2008). Initial and boundary conditions from the Geophysical Fluid Dynamics Laboratory (GFDL) hurricane forecasting model were used, along with WRF single-moment 6-class (WSM-6) microphysics (Hong and Lim 2006), the Yonsei University (YSU) planetary boundary layer (Noh et al. 2003; Hong et al. 2006), the Goddard shortwave (Chou et al. 1998),
and Rapid Radiative Transfer Model (RRTM) long-wave (Mlawer et al. 1997) parameterization schemes. Figure 1b shows a convective–stratiform partitioning applied at the same time as in Fig. 1a by using the method described in Braun et al. (2010), which is similar to that by Tao et al. (1993). Convection outside the tropical cyclone core appears mostly convective in the upwind region of a spiral rainband (east and southeast of the core) while an extensive region of stratiform convection is present to the north of the core. Although different types of convection and anomalies (e.g., environment induced and storm-motion induced) coexist in tropical cyclones outside the core, vorticity dipoles of different size and strength appear to be the dominant features of the convection-induced vorticity field. These vorticity dipoles are generated through the tilting of horizontal vorticity into the vertical direction by updrafts embedded within convection (e.g., Fig. 19 of Powell 1990; Fig. 10 of Montgomery et al. 2006).

2. Numerical model and initial conditions

In this study, we use a fully nonlinear model of two-dimensional, incompressible fluid to examine how a stronger, larger tropical cyclone core vortex interacts with convection-induced small vorticity dipoles whose magnitude is comparable to that of the high-vorticity tropical cyclone core. The numerical model is a spectral element model presented by Iskandarani et al. (1995) and Iskandarani (2008). The domain is a 400 km × 400 km box that has periodic boundary conditions in the zonal direction and is discretized into 80 × 80 spectral elements of 5-km size. An eighth degree polynomial is used to approximate the solution in each element, yielding an average effective grid spacing of 625 m. A 3-s time step is used to integrate for 12 h with a third-order Runge–Kutta scheme. A low value of viscosity (5 m² s⁻¹) is used.

The high-vorticity core vortex is represented by a smoothed modified Rankine profile:

\[
\zeta(r) = \begin{cases} 
\xi_{\text{core}}r, & 0 \leq r \leq r_{\text{core}} - d, \\
\xi_{\text{core}}S\left(\frac{r - r_{\text{core}} + d}{2d}\right) + 0.5\xi_{\text{core}}(1 - \alpha)\left(r - r_{\text{core}}\right)^{-1(1+\alpha)}, & r_{\text{core}} - d \leq r \leq r_{\text{core}} + d, \\
0.5\xi_{\text{core}}(1 - \alpha)\left(r - r_{\text{core}}\right)^{-1(1+\alpha)}, & r \geq r_{\text{core}} + d,
\end{cases}
\]

where \(r\) is the radial distance from the center of the core vortex, \(r_{\text{core}}\) is the radial width of the uniform vorticity region in the center of the modified Rankine profile, \(\xi_{\text{core}}\) is the value of the uniform vorticity, and \(\alpha\) is the decay parameter. The cubic Hermite polynomial, \(S(x) = 1 - 3x^2 + 2x^3\), that satisfies \(S(0) = 1, S(1) = 0, S'(0) = S'(1) = 0\), is used over the transition zone of \(2d\) to smooth the sharp vorticity decrease outside \(r_{\text{core}}\). Also \(\xi_{\text{core}} = 0.008\) s⁻¹, \(r_{\text{core}} = 15\) km, \(\alpha = 0.5\), \(d = 5\) km are used in (1), making the peak wind speed equal to 56.2 m s⁻¹ at a radius of maximum wind (RMW) of 16.6 km (see Fig. 2a). This is in good agreement with observations that the average intensity around the time of peak intensity at approximately \(r = 1.38a\).

3. Results

a. A like-signed convection-induced vortex

We first reproduce the formation of a concentric vorticity ring structure from the interaction between a tropical cyclone core vortex and a like-signed, convection-induced vortex. The core vortex is initially centered at \(x = -60\) km and \(y = 0\) km. The convection-induced vortex is created by using (2) with \(\xi_{\text{dipole}} = 0.002\) s⁻¹ and \(a = 45\) km and is initially centered at \(x = 60\) km and \(y = 0\) km (see Fig. 2b). For this section only, the model is integrated for 24 h. As briefly described in section 1, the core vortex shears apart the convection-induced vortex and turns it into thin filaments that wrap around the core vortex (Fig. 2c). Vorticity filaments then become barotropically unstable (Fig. 2d) and break down and mix to form a concentric ringlike structure of considerable width (Fig. 2e). This vorticity ring is stabilized by the core vortex-induced differential rotation, which opposes the self-imposed differential rotation of the ring and prevents phase locking between the ring edges (Kossin et al. 2000). This concentric ring
structure is clearly separated from the high-vorticity core by a region of lower vorticity (Fig. 2f), and it is clearly associated with a secondary wind maximum (dashed line in Fig. 2a).

b. A single vorticity dipole

The interaction between the core vortex and the convection-induced vorticity field as represented by a
FIG. 3. Vorticity field (×10⁻³ s⁻¹) from the interaction between the core vortex and a positive–negative vorticity dipole at (from top to bottom) t = 0, 2, 4, and 6 h. See section 3b for details.
FIG. 4. Vorticity field ($10^{-1} \text{s}^{-1}$) from the interaction between the core vortex and multiple vorticity dipoles at (from left to right) $t = 0, 3, 6,$ and $12$ h. See section 3c for details.
positive–negative vorticity dipole is now examined. Hereafter, the core vortex is initially centered at \( x = 0 \) km and \( y = 0 \) km, and \( \zeta_{\text{dipole}} = 0.004 \) s\(^{-1} \) and \( a = 5 \) km are used in (2) to create a vorticity dipole. In the left column of Fig. 3, the negative and positive vorticity patches are initially centered at \( x = 50 \) and \( 65 \) km and \( y = 0 \) km, respectively, while the right column of Fig. 3 has them initially centered at \( x = 50 \) km and \( y = \pm 7.5 \) km, respectively. The separation distance between the patches is about 3 km. The main difference between these two configurations is the orientation of the initial self-induced motion of the dipole with the circular advection by the core vortex. Because the separation distance between the patches of the dipole is small, both the positive and negative vorticity patches induce considerable cyclonic and anticyclonic flows on their respective counterparts. The resulting self-induced motion of the dipole is equal to the induced flow of each vorticity patch on the other (Batchelor 1967, chapter 7.3).

The self-induced motion is initially directed parallel to the circular advection in the left column of Fig. 3 but perpendicular to the circular advection in the right column of Fig. 3. The core vortex again completely shears apart the positive vorticity patch and turns it into filaments. However, the negative vorticity patch continuously interacts with positive vorticity filaments to prevent them from forming a coherent ringlike structure. Vorticity filaments gradually decay because of diffusion. In both cases as well as their permutations that have the self-induced motion in the opposite direction, no concentric ringlike structure is formed.

c. Multiple vorticity dipoles

There is a large amount of convection around the core of tropical cyclones, and within convection exist numerous updrafts and downdrafts. Therefore, a more realistic representation of the convection-induced vorticity field is to use multiple vorticity dipoles. Three cases are shown. The top row of Fig. 4 is initialized with four groups of two of the same vorticity dipole used in Fig. 3. The middle row of Fig. 4 is initialized with a spiral band of vorticity dipoles, as observations (e.g., Senn and Hiser 1959; Barnes et al. 1983) show that convection outside the core of tropical cyclones is typically organized in spiral shapes. The radially inner vorticity patches spiral radially inward from \( r = 65 \) to 50 km in the counterclockwise direction while the outer vorticity patches spiral radially inward from \( r = 80 \) to 65 km. The separation distance among the vorticity patches is 3 km. The bottom row of Fig. 4 is initialized with the same spiral band of vorticity dipoles used in the middle row of Fig. 4 on top of a spiral band of weak positive vorticity. This spiral band of weak positive vorticity represents the enhanced background vorticity from stretching driven by net condensational heating within a spiral band. The spiral band of weak positive vorticity is constructed from 50 overlapping circular patches of positive vorticity, similar to the method in which spiral band diabatic heating was constructed in Moon and Nolan (2010). Each circular patch of positive vorticity is created by using (2) with \( \zeta_{\text{dipole}} = 0.00015 \) s\(^{-1} \) and \( a = 20 \) km, and its center location spirals radially inward from \( r = 72.5 \) to 57.6 km in the counterclockwise direction.

In all cases described above, the high-vorticity core vortex shears apart the like-signed positive vorticity patches of the dipoles and turns them into filaments that wrap around the core vortex. However, negative vorticity patches continuously interact with positive vorticity filaments to prevent them from forming a coherent concentric ringlike structure. It is possible to argue that a thin concentric vorticity ring structure is formed for all cases at \( t = 6 \) h (third column of Fig. 4) and 12 h (fourth column of Fig. 4). However, Fig. 5 confirms that the azimuthally-averaged tangential velocity field around the center of the core vortex does not exhibit secondary wind maxima that could be associated with secondary eyewall formation in tropical cyclones. Just as K04 stated for the DW92 results, the concentric ring is too thin to be identified as representative of secondary eyewall formation.

4. Summary and discussion

The formation of a concentric vorticity ring structure from the interaction between the high-vorticity core vortex and the positive–negative vorticity dipoles has been examined in the framework of two-dimensional, incompressible flow. In contrast to previous studies, the vorticity field induced by peripheral moist convection outside the rapidly rotating tropical cyclone core was represented by vorticity dipoles that have half the magnitude of the core vortex but are much smaller. This representation is more consistent with observations and full-physics simulations of tropical cyclones. In addition to the cases shown in section 3, many other combinations of vorticity dipoles were examined. However, in all cases considered in this study, no coherent concentric ringlike structure of vorticity was formed.

The favorable results from previous studies using two-dimensional, incompressible flow to study secondary eyewall formation in tropical cyclones suggested that the essential mechanisms of the phenomenon could be fully captured by two-dimensional dynamics. The effects of convection were represented as two-dimensional vorticity perturbations in this approach. If the above mechanisms are valid, then the concentric vorticity ring structure should be produced as the end state of two-dimensional
numerical simulations over a large parameter space, as a large fraction (~60%–70%) of tropical cyclones of sufficient strength (i.e., maximum wind speed of ~60 m s$^{-1}$ or higher) are observed to form secondary eyewalls at some time during their lifetime (Kossin and Sitkowski 2009; Kuo et al. 2009). The lack of a coherent concentric vorticity ring in the final state when vorticity dipoles are used as initial conditions indicates that the essential mechanisms of secondary eyewall formation are not fully captured by the two-dimensional framework. The results of this study suggest that the axisymmetrization process under the simplification of two-dimensional incompressible flow is incomplete for describing secondary eyewall formation. Recent observational and numerical simulation studies (e.g., Houze et al. 2007; Terwey and Montgomery 2008; Houze 2010; Judt and Chen 2010) emphasize the importance of three-dimensional processes in secondary eyewall formation.

Acknowledgments. Support from the National Science Foundation through Grants ATM-0756308 for D. S. Nolan and Y. Moon and OCE-0622662 for M. Iskandarani is gratefully acknowledged. Constructive reviews by Drs. Jim Kossin, Shigeo Yoden, and two anonymous reviewers and helpful comments from Dr. Brian E. Mapes are appreciated. We thank Mr. Munchiko Yamaguchi for help with initially setting up the numerical model. Calculations were performed on the High Performance Computing core at the Center for Computation Science of the University of Miami.

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