The sea-surface structure of North Brazil Current
Rings derived from shipboard and moored ADCP
observations

G. P. Castelão
Division of Meteorology and Physical Oceanography of Rosenstiel School of
Marine and Atmospheric Sciences, University of Miami

W. E. Johns
Division of Meteorology and Physical Oceanography of Rosenstiel School of
Marine and Atmospheric Sciences, University of Miami

G. P. Castelão, Division of Meteorology and Physical Oceanography of Rosenstiel School of
Marine and Atmospheric Sciences, University of Miami, 4600 Rickenbacker Causeway, Miami,
FL 33149, USA. (guilherme@castelao.net)

W. E. Johns, Division of Meteorology and Physical Oceanography of Rosenstiel School of
Marine and Atmospheric Sciences, University of Miami, 4600 Rickenbacker Causeway, Miami,
FL 33149, USA. (wjohns@rsmas.miami.edu)
Abstract. In the western tropical Atlantic, the North Brazil Current retroflexion periodically sheds large anticyclonic rings, which then propagate northwestward. Between 1998–2000, the North Brazil Current Rings Experiment sampled a large number of these rings by shipboard and moored Acoustic Doppler Current Profiler. Ten of the sampled rings are analyzed in this study, focusing on their sea surface dynamic properties. The rings exhibit a radial structure consisting of two regimes, an “inner” core region in near solid body rotation, and an “outer” ring regime with an approximately exponentially decaying structure. The observations show a sharp change in vorticity at the regime transition and the presence of a strong opposite vorticity shield bounding the inner solid body core. We show that Gaussian models, commonly used to represent the surface expression of these and other rings, are adequate for determining the sea surface height anomaly, but tend to poorly estimate other properties such as the maximum swirl velocity. Therefore we propose a new two-part model as a better approximation of the rings’ radial structure. According to the cyclo-geostrophic balance approximation, the sea surface height distribution across the inner ring has a parabolic shape, while the outer ring has an exponential structure similar to the velocity field. Interestingly, many of the observed rings have an intensity very close to the theoretical limit for anticyclones at these latitudes, which is believed to be due to inertial instability.
1. Introduction

In the equatorial south Atlantic, the South Equatorial Current divides into a northward flow and a southward flow at the western boundary. The southern branch (sSEC) feeds the southward flowing Brazil Current (BC), while the central branch (cSEC) merges with the northern branch of the South Equatorial Current (nSEC) to feed the North Brazil Current (NBC) (Figure 1) [Cochrane et al., 1979; Schott et al., 1998]. Near 7°N the NBC separates from the coast and retroflects eastward, feeding the North Equatorial Counter Current (NECC) at mean latitude of 6.6°N ±2.0° [Fonseca et al., 2004]. The retroflection shows a seasonal cycle related to the migration of the Inter Tropical Convergence Zone (ITCZ), and also interannual variability [Fonseca et al., 2004].

It has been known for almost two decades that the retroflection periodically pinches off on itself, shedding rings that move toward the northwest [Johns et al., 1990; Didden and Schott, 1993]. The NBC retroflection is typically well developed between June to February, while it is weak (or absent) between March to May [Johns et al., 1998]. Nevertheless, NBC rings (NBCR) are observed throughout the year [Goni and Johns, 2001; Johns et al., 2003], suggesting that another formation mechanism independent of the NBC retroflection also exists [Edwards and Pedlosky, 1998; Jochum and Malanotte-Rizzoli, 2003].

NBC rings have typical surface swirl velocities of 1 m/s, radii of maximum velocity of 100–160 km, an overall size of approximately 400 km, and a maximum sea surface height anomaly (SSHA) of 5–30 cm [Richardson et al., 1994; Goni and Johns, 2001; Wilson et al., 2002; Goni and Johns, 2003; Johns et al., 2003]. Goni and Johns [2001] observed 34 rings between 1992–1998 using TOPEX/Poseidon data, which moved along trajectories nearly
parallel to the 500m isobath with a mean translation speed of 14 km/day, and an average formation rate of NBC rings of once every 2 months.

Using a numerical ocean circulation model with an imposed 14 Sv Meridional Overturning Cell (MOC), Fratantoni et al. [2000] describe three major pathways for Atlantic inter-hemispheric exchange: 6 Sv through a frictional western boundary current, 4.2 Sv by wind forced equatorial upwelling with subsequent northward transport in the Ekman layer, and approximately 3.0 Sv carried by an average of 3–4 NBC rings per year. Talley et al. [2003] suggested a stronger MOC of 18 Sv, and observations suggest a higher ring shedding rate as well. Goni and Johns [2003] found a ring-shedding rate of 3–7 rings per year using altimetric data while Johns et al. [2003], using ship survey and mooring data, describe 8–9 rings per year, which include sub-surface rings with weak or absent surface signals. Johns et al. [2003] found an annualized transport of 9.3 Sv by rings, which is a considerably higher contribution by the rings to the MOC return flow than previously thought.

Garraffo et al. [2003] modeled the NBC ring generation process and found several different types of rings, similar to those observed, that they classified as shallow, intermediate, deep, and sub-surface rings. If the subsurface rings, which showed a weak or almost nonexistent surface signal, were not counted, a similar ring generation rate to that obtained by Goni and Johns [2003] with altimetric data was obtained. The model suggested an average southern hemisphere water transport of 7.5 Sv, representing approximately 40% of the total northward interhemispheric transport in the model between the surface and intermediate water layers. Both models and observations therefore indicate that NBC
rings are an important process for inter-hemispheric transport of mass and heat in the Atlantic.

In this paper we use detailed in situ data from the 1998–2000 North Brazil Current Rings Experiment (NBCRE) to examine the internal structure and dynamic properties of a number of NBC Rings. Our focus is on the surface properties of the rings, that is, their surface velocity and vorticity structure, and their implied sea surface height structure, which are then compared to altimetric data. The availability of high quality direct measurements of several NBC rings provides an opportunity to better understand their dynamics, and to evaluate the considerations and approximations used in previous studies. Towards that, this study characterizes the typical scales of the sampled rings and defines adequate approximations to better represent their dynamics. We develop a piecewise model for the radial velocity structure of the rings, from which the vorticity structure and SSH anomaly of the rings are reconstructed, and compared with other models commonly used for oceanic rings.

2. Data

This study uses two different in situ datasets, from shipboard surveys and moored time series [Wilson et al., 2002; Johns et al., 2003]. Four rings were observed between 6°–10°N from the NBCRE cruises, conducted during: November–December 1998 (one ring, R–1), February–March 1999 (two rings, R–2 and R–3), and June 2000 (one ring, R–4). The current profile was continuously sampled using a hull-mounted 150kHz narrowband shipboard Acoustic Doppler Current Profiler (ADCP). The measurements were binned in 10 km segments along 2 or 3 transects per ring, attempting to cross as close as possible to the ring center.
Of the 4 rings surveyed, three are analyzed here: ring R–2, a deep-reaching ring with a velocity structure reaching 2000 m, R–3, a shallow ring confined to within 200m of the surface, and R–1, a subsurface ring with the velocity core below the thermocline and almost no sea-surface signal. R–1 was, therefore, not included in the sea surface height analysis, and ring R–4 was excluded due to limited spatial sampling.

The second dataset was obtained by a mooring deployed near 9°N 53°W, from November 1998 to June 2000, composed of 8 conventional current meters (Vector Averaging Current Meters; VACMs) and an ADCP mounted on the top buoy at 250m looking upward [Johns et al., 2003]. The water properties were registered by a second mooring 2 km away with a vertical array of T–S recorders. These moorings recorded the passage of 11 surface rings, designated here as M–1 to M–11, seven of which are included in this study. Of the four excluded rings, one case suggested an interaction between two rings, and the others did not have their radial structure well sampled since they passed too far away from the mooring. The nearest surface velocity in each ring was taking as the vertical mean of the 3 shallowest bins, 22, 30 and 38m for the ship survey data and 20, 30 and 40m for the mooring dataset.

Finally, we use an altimetric data set composed of the along-track measurements from TOPEX/Poseidon plus the combined mean dynamic topography Rio05 [Rio and Hernan-dez, 2004], which provide along-track absolute dynamic topography for comparison with the in-situ results.

3. Methodology

We analyze the rings in the framework of a cylindrical coordinate system, in radial ($v_r$) and azimuthal ($v_\theta$) components, where the velocity is defined by $\vec{V} = v_r \hat{r} + v_\theta \hat{\theta}$. This is
similar to the natural coordinate system used by Holton [2004], but we use \( r > 0 \) and \( v_{\theta} < 0 \), since it is more intuitive. For this decomposition, the relation \( \vec{V}(r) \) must be established, which requires an accurate estimate of the ring center.

For a perfectly axisymmetric, non–translating ring, all the flow should be contained in the azimuthal component, \( v_{\theta} \). Therefore, to find the best estimate of the ring center \((x_c, y_c)\), a nonlinear least–squares (Gauss–Newton method) optimization was applied that minimizes the difference between the observed speed magnitude (\( |\vec{V}| \)) at all survey points and the azimuthal component of the observed velocities in a cylindrical coordinate system with origin at \( x_c, y_c \). This is written as:

\[
|\vec{V}| = -u \sin(\theta) + v \cos(\theta) + \epsilon 
\]

\[
\theta = \tan^{-1}(y_{r}/x_{r}) 
\]

\[
x_{r} = x - x_{c} 
\]

\[
y_{r} = y - y_{c} 
\]

where \( u \) and \( v \) are the velocity components observed, \( |\vec{V}| \) is the total magnitude observed, \( x \) and \( y \) are the position of the samples, and \( \epsilon \) is the residual to be minimized.

For a translating ring, the velocity field \((u, v)\), as well the sample locations \((x, y)\), need to be corrected to isolate the rotational part of the ring flow and to account for the movement of the ring over the course of a ring survey, which typically takes several days (see Fig. 2). Therefore the model becomes:

\[
|\vec{V}| = -(u - u_{c}) \sin(\theta) + (v - v_{c}) \cos(\theta) + \epsilon 
\]

\[
\theta = \tan^{-1}(y_{r}/x_{r}) 
\]

\[
x_{r} = x - x_{c} - u_{c}t 
\]
\[ y_r = y - y_c - v_c t \]

where \( u_c, v_c \) are the zonal and meridional components of the ring translation velocity and \( t \) is the time relative to the median date/time of the samples collected in each ring.

This method was applied to the ship surveyed dataset, where the parameters \( x_c, y_c, u_c \) and \( v_c \) are all determined as part of the least-squares minimization. For the mooring data, which provide only one “slice” through each ring, there is not enough data to estimate all four parameters, and for these rings the ring translation speed \((u_c, v_c)\) was estimated independently from an array of Inverted Echo Sounders (IES) that was deployed concurrently with the moorings \([\text{Garzoli et al.}, 2003, 2004]\).

The mooring dataset is actually composed of a time series of the vector velocity as the rings pass through the mooring location. With a constant propagation speed, the mooring time series data can be transformed into a “virtual” section through each ring, as described in detail in \(\text{Johns et al.} [2003]\). Using the translation speeds derived from the IES array, each ring passage was converted into an equivalent spatial section, and these sections were treated in the same way as the ship surveyed sections, except that only the ring center location \((x_c, y_c)\) was determined from the minimization (see Fig. 3).

Failure to account for the ring translation speed can result in errors in the ring center estimate of up to 30 km, for typical ring sizes and propagation speeds, leading to considerable errors in the cylindrical decomposition of the ring and its radial velocity structure.

For the initial estimate of the ring center, all the survey data available for each ring are used. Due the interaction of the ring with the external environment, velocities near the edge of the ring are relatively noisy, which can adversely impact the determination of the
ring center. Therefore, to refine the estimate of the ring center, data collected at the outer edges of the ring are progressively excluded in an iterative procedure. From the initial fit we obtain an estimate of the radius ($R_{\text{max}}$) at which the maximum velocity ($V_{\text{max}}$) is observed. On the second and successive iterations, the fitting process is limited to the data inside $1.25R_{\text{max}}$, and continued until it converges, resulting in the final estimate of the ring center (and also the ring translation velocity, for the ship surveyed rings). The determination of ring center is illustrated in Figure 2, where the original and the translation–corrected data for ring R–2 are shown. The equivalent procedure for the mooring M–4 is illustrated in the Figure 3.

Once the ring center is determined, each section (one section for each mooring–sampled ring and 2–3 for the ship surveyed rings) is split in two separate radial sections, and individually analyzed. Then, all available radial sections for each ring are merged together in the subsequent analysis for each ring.

4. Results

4.1. Ring Velocity Structure

To estimate the radius of maximum velocity ($R_{\text{max}}$) and maximum swirl velocity ($V_{\text{max}}$) for each ring, the observed ($v_\theta$, $r$) data are first smoothed using a cubic spline with one degree of freedom for every 15 km along the radius. The azimuthal velocity is then interpolated to 5 km resolution, and the maximum velocity on this curve is defined as $V_{\text{max}}$, and its respective position $R_{\text{max}}$.

Figure 4 shows an example for ring R–2, where the individual radial sections, as well as the composite estimate of $V_{\text{max}}$ and $R_{\text{max}}$, are illustrated. As this figure makes clear, there can be significant asymmetries in the rings, resulting in differing estimates for $V_{\text{max}}$ and
$R_{\text{max}}$ within a given ring. The mean $V_{\text{max}}$ and $R_{\text{max}}$ for each ring are taken as the average of the individual ring section estimates, and the uncertainty as the respective standard deviation. For the sampled rings, the $V_{\text{max}}$ values ranged from 0.65 to 1.16 m/s and $R_{\text{max}}$ values from 85 to 158 km (Table 1).

To be able to compare the structure of all rings together, the velocities and radial positions were normalized by their respective $R_{\text{max}}$ and $V_{\text{max}}$ values (Fig. 5). The resulting non-dimensional structure has two regimes, separated by $R_{\text{max}}$. The central core of the ring, here called the "inner ring", shows a solid body rotation pattern, hence with the velocity linearly dependent on the radius, i.e. $v_\theta = rV_{\text{max}}/R_{\text{max}}$. The inner part of all the rings together, normalized by their $R_{\text{max}}$ and $V_{\text{max}}$, show a linear regression with $r^2 = 0.75$.

The regime transition at the edge of the inner ring is usually sharp. The surrounding annulus, here called the “outer ring”, has a radially–decaying structure suggestive of either an exponential or inverse power law decay. Fitting an exponential structure of the form $v_\theta(r) = V_{\text{max}} \exp\left[-(r - R_{\text{max}})/\lambda\right]$ yields decay scales for the outer ring regime of 35 to 107 km (Table 1), while an inverse power law, of the form $v_\theta(r) = V_{\text{max}}(R_{\text{max}}/r)^n$ yields exponents ranging from 1.5 to 4. A Rankine vortex, which rotates like a solid body, would be the equivalent to $n = 1$. Both forms fit the outer ring structure fairly well. An exponential outer ring structure was also suggested by Olson [1980, 1991] for Gulf Stream rings, and used in oceanic vortex modeling studies [Masina and Pinardi, 1993; Valcke and Verron, 1997], and so will be used in what follows.

There is considerable variability in the outer ring decay scale for different rings. Moreover, there is no apparent relationship between the outer ring decay scale and the size ($R_{\text{max}}$) of the inner ring, that is, the inner and outer ring dimensions are not self similar.
We therefore propose as the best model for the azimuthal velocity component of the rings the piecewise model defined by:

\[
v_\theta(r) = \begin{cases} r \frac{V_{\text{max}}}{R_{\text{max}}} & \text{for } r \leq R_{\text{max}} \\ V_{\text{max}} \exp \left( -\frac{r-R_{\text{max}}}{\lambda} \right) & \text{for } r > R_{\text{max}} \end{cases}
\]  

(3)

The outer ring structure of all rings together, normalized by their individual e-folding scales (\(\lambda\)), is presented in Figure 5 (right panel). The average value and standard error of the rings is \(\lambda = 58 \pm 22\) km, again indicating the large variability in this parameter from ring to ring. This contrasts with the oceanic ring model of Valcke and Verron [1997], in which \(\lambda\) was scaled as \(1/3 R_{\text{max}}\). However, the median value of \(\lambda\) in our observations (48 km), which is more robust to outliers, is about one third of the median value of \(R_{\text{max}}\) (136 km), consistent with the ratio of these scales chosen by Valcke and Verron [1997].

4.2. Ring Vorticity Structure

The vertical component of the relative vorticity \((\omega_z)\) in the cylindrical coordinate system, neglecting the radial flow, is given by

\[
\omega_z = \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta).
\]

(4)

For each ring, \(\omega_z\) was estimated along each radial section from the smoothed velocity structure. The vorticity structure of rings R–1 and R–2 are illustrated in Figure 6. Due to the sampling methodology, the extremes of the radii – close to the center and on the border of the ring – can be better observed in the shipsurveyed rings. As expected for a solid body, from equation 4, the inner ring regime has an approximately constant vorticity close to \(\omega_0 = 2V_{\text{max}}/R_{\text{max}}\). Surrounding the inner ring is observed an annulus of opposite vorticity, typically referred to as a vorticity “shield” [Saffman, 1995], whose strength depends on the swiftness of the outer velocity decay (i.e., \(\lambda\)). Due to the discontinuous
first derivative of $v_{\theta}(r)$ at $R_{\text{max}}$ in our two–part model, the consequent vorticity profile has a jump to values of opposite sign at $R_{\text{max}}$, whereas the actual transition for rings is continuous – but still usually very sharp.

The associated two-part vorticity model for the rings is:

$$
\omega_z(r) = \begin{cases} 
2 \frac{V_{\text{max}}}{R_{\text{max}}} & \text{for } r \leq R_{\text{max}} \\
V_{\text{max}} \left( \frac{1}{r} - \frac{1}{\lambda} \right) \exp \left( -\frac{r-R_{\text{max}}}{\lambda} \right) & \text{for } r > R_{\text{max}}
\end{cases}
$$

(5)

Values of the inner core vorticity ($\omega_0$), obtained from a robust linear regression on the inner ring, and the maximum vorticity in the outer shield ($\omega_s$), are given in Table 1. The $\omega_s$ is usually smaller than $\omega_0$, but the same order of magnitude. On average $\omega_s$ is half the size of $\omega_0$, with opposite sign.

The evolution and interaction of vortices has been numerically studied \[\text{[Carton and Legras, 1994; Higgins et al., 2002]}\] considering a generalized velocity structure:

$$
v_{\theta}(r) = \frac{1}{2} \omega_{\theta} r \exp \left( -\left( \frac{r}{\delta} \right)^{\alpha} \right),
$$

and an associated vorticity structure of:

$$
\omega(r) = \omega_0 \left[ 1 - \frac{1}{2} \alpha \left( \frac{r}{\delta} \right)^{\alpha} \right] \exp \left[ -\left( \frac{r}{\delta} \right)^{\alpha} \right]
$$

hereafter referred as the CL94 model.

This model shows a reasonable overall fit to our ring data as well (Fig. 6), but differs from our chosen model in prescribing a self–similar scaling of the inner and outer core dimensions, which we do not find to be the case for NBC rings. This model also tends to underestimate the $V_{\text{max}}$, and has a smoother transition of the vorticity field at $R_{\text{max}}$, producing a vorticity shield that is wider and less intense than observed.

The NBC rings observed here have, therefore, a more intense but narrower shield than the CL94 model, and a scale not linearly related to the inner core. Properly representing
the characteristics of the vorticity shield of oceanic rings is important, as this has been shown to have considerable bearing on their interaction and merger behavior [Carton, 1992; Masina and Pinardi, 1991; Verron and Valcke, 1994; Valcke and Verron, 1997].

We also calculate the Rossby number for each ring (Table 1), which compares the importance of the relative vorticity to the planetary vorticity, by the scale values. This number can be defined for the rings as [Olson, 1991],

$$Ro = \frac{V_{\text{max}}}{f_0 R_{\text{max}}}$$

where, by definition, the Ro is negative for anticyclones. The mean Ro number for the rings is $-0.33 \pm 0.07$, larger in magnitude than the $-0.19 \pm 0.07$ estimated by Fratantoni and Johns [1995]. The reason for this difference is that, while the $R_{\text{max}}$ scales are roughly the same in the two studies, the $V_{\text{max}}$ observed here are substantially higher.

4.3. The sea–surface height signal of NBC rings

To first order, the velocity field of the NBC Rings can be approximated by a cyclo–geostrophic balance, equivalent to the “regular high” case described by Holton [2004]. In this case the Coriolis acceleration is balanced by the pressure gradient force and centrifugal acceleration,

$$fv_\theta = g \frac{\partial \eta}{\partial r} - \frac{v_\theta^2}{r}$$  \hspace{1cm} (6)

where $r$ is the radius, $f$ the Coriolis parameter, $g$ the gravity and $\eta$ the SSHA (here again we have defined $r > 0$ and $v_\theta < 0$, for NBCR). With this relation, the sea surface structure of the ring can be estimated from the velocity structure.
The linear model of the velocity on the inner ring, applied to the gradient balance (eq. 6), leads to

\[
\frac{\omega_0}{4g}(\omega_0 + 2f)r = \frac{\partial \eta}{\partial r}
\]

Approximating the Coriolis parameter as a constant \((f_0)\) on the inner ring and integrating along the radius, the resulting \(\eta(r)\) is:

\[
\eta(r) = \eta_0 - ar^2
\]

where

\[
a = -\frac{\omega_0}{8g}(\omega_0 + 2f_0)
\]

and where \(\eta_0\) is the maximum elevation of the ring and \(a\) is constant. Therefore, the inner ring should have to first approximation a parabolic shape.

The outer ring is well approximated by an exponential decay (eq. 3), therefore, the model for the sea surface height field in NBC rings is piecewise with a parabolic “inner” ring structure and an exponential “outer” ring structure:

\[
\eta(r) = \begin{cases} 
\eta_0 - ar^2 & \text{for } r \leq R_{\text{max}} \\
\frac{V_{\text{max}}}{g} \left[ -\lambda f \exp\left(\frac{-(r-R_{\text{max}})}{\lambda}\right) + \frac{2R_{\text{max}}}{\lambda} \right] \exp\left(\frac{2R_{\text{max}}}{\lambda} \right) \text{Ei}\left(\frac{-2r}{\lambda}\right) & \text{for } r \geq R_{\text{max}}
\end{cases}
\]

where \(\text{Ei}(\cdot)\) is the Exponential Integral function.

The term including \(\text{Ei}(\cdot)\) accounts for the contribution of centrifugal acceleration to the sea surface height in the outer ring and is negligible compared with the first term. Neglecting this term, and equating the values of \(\eta(R_{\text{max}})\) at the junction of the inner and outer ring, \(\eta_0\) can be written as

\[
\eta_0 \approx aR_{\text{max}}^2 - \frac{V_{\text{max}}\lambda f}{g}
\]
or using the scale parameters,

\[ \eta_0 \approx -\frac{V_{\text{max}}}{2g} \left( V_{\text{max}} + f_0 R_{\text{max}} + 2\lambda f \right). \] (7)

where the \( f_0 \) came from the inner ring approximation and \( f \) from the outer ring. From the typical values obtained, it is expected a meridional asymmetry smaller than 3 cm.

The respective \( \eta_0 \) values estimated for each of the rings are listed in table 1, ranging from 9–38 cm, with a mean value of 22 cm. Only the radii which were sampled along at least one \( \lambda \) were considered.

It is interesting to note the extreme values of \( \eta_0 \) for R–3 and M–7. Both cases were surrounded by cyclones, therefore, surrounded by negative anomalies of the SSHA, which accentuate the SSH gradient and velocity at the edge of the ring. These are cases where the \( \eta_0 \) alone could mislead the evaluation of the strength of the rings. An alternative parameter to characterize the intensity of the ring is the SSHA only across the inner ring (\( \eta_{in} \)). That parameter is pertinent to both \( V_{\text{max}} \) and \( R_{\text{max}} \), is easy to estimate and is independent of the surroundings. On average, \( \eta_{in} \) accounts for about half of the total ring SSHA, except in cases like R–3 and M–7 where it is about one third of the \( \eta_0 \).

5. Discussion

The rings observed here are larger and stronger, with respect to their \( R_{\text{max}} \) and \( \eta_0 \), than the mean values estimated by Goni and Johns [2001, 2003] from altimetric data. Goni and Johns [2003] noted the possibility that their estimates could be biased low, since the satellite ground tracks did not always pass through the center of the rings. The \( V_{\text{max}} \) observed by Fratantoni and Johns [1995] are also smaller, and this difference can be explained by the difference in depth of observations. Fratantoni and Johns [1995] were
able to measure the velocity field only as shallow as 150m, while our measurements are for 30m. The datasets used in those studies are from different periods than analyzed here, hence this difference could possibly be due to different populations of rings being studied. Johns et al. [2003] showed, for the same mooring dataset used here, that the velocities of the identified rings at 50m depth were usually stronger than at 150m. The $R_{\text{max}}$ determined by Johns et al. [2003] for the rings in common with our study were within the standard deviation defined by our objective procedure. Fratantoni and Richardson [2006] also evaluated some of these same rings, but using drifters, and obtained relatively consistent $V_{\text{max}}$ estimates but slightly smaller than ours. This can be explained by the low probability of a drifter staying exactly at the rings’ $R_{\text{max}}$ for very long, and so their result is likely to be a lower bound.

Olson [1991] analyzed various rings observed around the world ocean, mostly from the subtropics, where the Ro of the anti–cyclones were typically close to -0.1, with only two observations with values lower than -0.2. The NBCR discussed here clearly stand out as strongly nonlinear features, with a mean Ro=-0.33 (Table 1).

We find that NBC rings have a strong relative vorticity “shield” around their inner cores, a feature which has important implications for ring–ring interactions and merger. NBC rings are shed in a quasi–regular fashion and often in close proximity to one another, and it may be expected that they often interact with each other as well as the South American continental margin.

Valcke and Verron [1997] concluded that vortices with shielded potential vorticity are very unlikely to merge with each other. When the surrounding annuli of two shielded vortices interact, they form lateral poles which repel the two vortex cores inhibiting the
merging. The NBCR observations indicate a narrower but stronger shield than they used in their theoretical model. We therefore expect a smaller critical distance of merging for the NBCR, i.e. they would be relatively “isolated” from each other on distances even closer than the $2.4R_{\text{max}}$ expected from Masina and Pinardi [1993]; Valcke and Verron [1997]. However, to more accurately predict the merger behavior of the NBCR, the potential vorticity structure, including the effects of stratification, needs to be considered rather than simply their near surface vorticity structure.

It is interesting to note that a number of the rings observed in this study are very close to the maximum strength attainable for anticyclones at this latitude. From eq. 6, the velocity is given by,

$$v_\theta = -\frac{fr}{2} \pm \sqrt{\left(\frac{fr}{2}\right)^2 + rg\frac{\partial \eta}{\partial r}}$$

(8)

where for an anticyclone ($\frac{\partial \eta}{\partial r} < 0$), a real solution is possible if

$$\left|\frac{\partial \eta}{\partial r}\right| \leq \frac{f^2r}{4g}$$

which defines a maximum slope possible for a certain radius and latitude. Figure 7 shows the calculated sea surface slope for each ring normalized by this theoretical limit. A number of the rings are close to or even at this limit, therefore, those rings have the largest negative pressure gradient allowed by the gradient balance, which occurs for $v_\theta \to -\frac{fr}{2}$. A section of ring R–3 is shown in detail where most of the inner ring is at this limit. The number of rings approaching this limit strongly suggests that there is a systematic process driving the rings toward this state.

One distinct possibility is that the rings are frequently subject to inertial instability. An extension for an $f$–plane approximation of the original Rayleigh’s instability analysis.
leads to a necessary and sufficient stability criterion for inertial instability of barotropic circular vortices in homogeneous fluids [Kloosterziel and van Heijst, 1991; Kloosterziel et al., 2007], defined as

$$\Phi = \frac{1}{r^3} \frac{dL^2}{dr} = \frac{2L}{r^2} \frac{dL}{dr} = \frac{2L}{r^2} Q$$  \hspace{1cm} (9)$$

where

$$L = v_\theta r + \frac{1}{2} f r^2$$ \hspace{1cm} (10)$$

is the absolute angular momentum, and

$$Q = \omega_z + f$$  \hspace{1cm} (11)$$

is the absolute vorticity. The fluid is stable if $\Phi \geq 0$, which is possible if the absolute angular momentum ($L$) and the absolute vorticity ($Q$) have the same sign.

Kloosterziel et al. [2007] studied the inertial instability of barotropic vortices by numerical simulations. They identified an unstable band on the inner core of anti-cyclones, where $L < 0$ and $Q > 0$. This instability, also known as centrifugal instability [Drazin and Reid, 2004; McWilliams, 2006], takes the form of successive overturning cells, alternating in sense of rotation along the fluid column, redistributing the angular momentum. A new stable state is achieved once the area of negative $L$ is removed. From equation 10, a marginally stable state of $L = 0$ will occur for $v_\theta = -fr/2$, which is identical to the pressure gradient limit for a gradient balance. For sufficiently small stratification and large Reynolds Number, Kloosterziel et al. [2007] showed as well that this instability and the subsequent stabilization process tend to propagate from the edge of the inner ring all the way to the vortex center. The new stable state is characterized by an inner core rotating like a solid body, but with a higher $R_{max}$ and lower $V_{max}$.
As expected from the discussion above, the number of rings observed in this study with a pressure gradient close to the theoretical limit is associated with their absolute angular momentum being close to 0 within the inner ring (Fig. 8, upper panel). Therefore, Rayleigh’s criterion for several rings indicates a marginally stable inner ring regime (Fig. 8, lower panel). The similarities with the observed rings and those simulated by Kloosterziel et al. [2007] suggests that the NBCR are commonly subject to inertial instability, which drives them to an inner ring rotating as a solid body at the upper limit of the gradient balance. Supporting this idea, Fratantoni and Richardson [2006] observed that surface drifters and RAFOS floats deployed in several radii of NBCR, gradually moved outward to a larger radii, but generally limited by $R_{\text{max}}$, which could be explained by the adjustment of the angular momentum by the inertial instability. Kloosterziel et al. [2007] also predict a sharp transition and steep velocity gradient at the edge of the inner ring, similar to our observations.

Some questions that remain open are – when does this instability first occur? Are some rings detached from the retroflection already in an unstable condition, and could the inertial instability be related to the pinch off process from the retroflection itself? Or does a tendency for potential vorticity conservation in the rings, as they migrate northward, cause changes in their velocity structure that lead to the onset of the instability? Unfortunately our data set provides no information on the temporal evolution of the rings, and is therefore unable to answer these questions. High-resolution ocean numerical models could perhaps provide further insight into this process.

Finally, it is worthwhile to discuss how ring parameters derived from the proposed piecewise model compare to those derived from other common models. Most previ-
analysis of oceanic rings based on altimetric data use a Gaussian model, $\eta(r) = \eta_0 \exp[-r^2/(2R_{\text{max}}^2)]$, to represent the rings structure [Goni and Johns, 2001, 2003]. An advantage of this approach is that a simple two parameter ($\eta, R_{\text{max}}$) model can fully describe the structure. However, it is useful to examine the consequences of this assumption. Therefore, for each ring, the Gaussian model was fit to the SSHA derived from application of the cyclo–geostrophic balance to the observed velocity structure. New $R_{\text{max}}, V_{\text{max}}$ and $\eta_0$ were then obtained from the parameters of the Gaussian model, and compared to those for the piecewise model.

For our observed range of parameters, the overall shape of the SSHA is reasonably well represented by the Gaussian model. There is a tendency to over–estimate the $\eta_0$ by at most 5%, which means less then 1 cm. That is less than the uncertainty of the satellite altimetric data. The radius of maximum velocity is, on average, the same, with a standard deviation of 25 km.

The largest discrepancy occurs for the swirl velocity. Since the outer–ring decay scale ($\lambda$) is not systematically related to either $R_{\text{max}}$ or $V_{\text{max}}$, the Gaussian model with only 2 degrees of freedom cannot well represent all the observed cases. Furthermore, due to the number of rings close to the cyclo–geostrophic balance limit, the sea level slopes derived from the Gaussian fit exceed, in several cases, the theoretical intensity limit resulting in unphysical (imaginary) velocities.

Figure 9 shows an example of this for a radial section of ring R–2. Despite the relatively good fit of the Gaussian profile to the observed SSHA (Figure 9A), the velocity structure estimated by applying the cyclo–geostrophic balance to the Gaussian fit is quite different from the observed velocity structure. In this case the $R_{\text{max}}$ from the Gaussian fit param-
eters underestimated the real $R_{max}$ by 18 km, and underestimated $V_{max}$ by 26 cm/s. Of the 26 radial sections studied, 14 had an unphysical velocity solution for the Gaussian SSHA fit.

The Gaussian model was also evaluated against synthetic rings, created from the proposed piecewise model, using the scale parameters observed (Table 1). In these tests the estimated velocity from the Gaussian is underestimated by as much as 45%. Analyzing the synthetic cases, it is clear that the Gaussian model is not able to represent the fast transition between the inner and the outer ring. Olson [1980] also noted limitations of the Gaussian model on Gulf Stream rings.

Independent of the fitting model, the along track altimetric data has an intrinsic limitation. In general, the altimetric section will not cross exactly through the ring center, therefore, there is a tendency to underestimate the maximum elevation and the sea surface slopes (Fig. 10). The moored ADCP sections did not coincide with the ring center either, but the velocity field allows estimation of a ring center position not along the sampled track, and so defines a corrected radial position. Such an approach is not possible using the along track altimetric data.

We conclude, therefore, that the Gaussian model provides a good approximation of the ring sea-surface height signature. However, the Gaussian model should not be used to estimate the velocity structure of the rings, as it can produce inaccurate (and sometimes unphysical) estimates of $V_{max}$.

A suggested approach for future altimetric analysis of NBC Rings is to use a Gaussian fit to define the approximate inner/outer ring boundary ($R_{max}$). Once the inner core and the outer ring are defined, the proposed piecewise model (eq. 3) is fitted, leading to $a$ and
so $\omega_0$ and $V_{\text{max}}$. To evaluate this technique, the rings R–2 and R–3 were re–analyzed using TOPEX data. The translation velocity of the rings were considered to determine the best altimeter passes. The SSHA derived from the altimeter are similar to those estimated from the velocity fields, but, as predicted, the altimeter tended to underestimate the rings intensity. The $R_{\text{max}}$ was underestimated by 28 km for R–2 and 30 km for the R–3, while the $V_{\text{max}}$ was underestimated by 21 and 10 cm/s, respectively.

The main core for some NBCR can be well below the subsurface, with small, if not negligible, surface signal. However, these are four times less frequent then the surface intensified rings [Johns et al., 2003; Garraffo et al., 2003]. It has been proposed that the near surface and sub–surface rings are independent features that can in sometimes couple to result in a deep reaching ring, or cleave apart to produce separate surface and sub–surface coherent eddy features [Jochum and Malanotte-Rizzoli, 2003; Garraffo et al., 2003; Fratantoni and Richardson, 2006]. Therefore, to infer in a robust way the vertical structure of rings at a certain time only using only the sea surface signal is, if feasible, not a simple task. The satellite altimetry has the potential to provide not only a NBCR climatology but also a monitoring system of changes in NBCR properties with time. With that in mind, we simplify our development focusing on the near surface intensified rings.

6. Summary and Conclusions

This study is the first to closely examine the detailed velocity and vorticity structure of NBC rings, and their associated SSHA structure. An objective method to define the ring center, based on the minimization of the velocity residual for an axis–symmetric ring, was implemented showing good results, and found to be robust enough to be applied as an automated procedure. With a sufficient amount of data collected in a ring, both the ring
center and translation speed can be estimated. Although not discussed earlier, we found that an additional minor improvement of the ring center estimate could be obtained if a solid body inner core structure was imposed in the fitting process, i.e., incorporated in the expected model. However, this approach was not used here because it takes as an assumption one of the main ring properties we intended to investigate.

We find the best overall model for the surface velocity structure of NBC rings is a two-regime, piecewise model of the form specified in equation 3, increasing linearly with radius inside of $R_{\text{max}}$ and exponentially decaying outside of it. A linear relation between velocity and the radius on the inner ring leads to a SSHA with a parabolic shape to first approximation.

Since a relation between $R_{\text{max}}$ and the outer decay scale $\lambda$ could not be identified, we conclude that those scales have considerable independence from each other. Therefore, any model that does not allow for that will be limited in its ability to correctly describe the range of ring geometries observed.

The rings showed a solid body rotation pattern in the inner ring, therefore a constant (negative) relative vorticity. This inner core is surrounded by a shield of opposite (positive) vorticity, stronger than that contained in most models used for theoretical studies of ocean vortices. The continuous model used by Carton and Legras [1994] is a reasonably good approximation of the velocity structure of the NBC Rings, but tends to under-estimate the maximum velocity and smooth the transition from the solid body to the shield, which could determine different characteristics regarding the interaction between NBC Rings.

The gradient balance determines a theoretical limit for the ring’s velocity, and so for the maximum SSHA slope in the ring. The fact that many of the rings show a velocity near this
limit suggests that they are subject to inertial (centrifugal) instability, whose onset, and subsequent stabilization of the rings, occurs at this theoretical limit. After redistributing the absolute angular momentum, a new stable state is reached, which explains why the inner core rotates like a solid body with $v_\theta \rightarrow -fr/2$. Whether this instability is occurring immediately after (or during) ring formation, or develops later as they migrate northward along the western boundary, is unknown.

A Gaussian model, commonly used to represent the structure of ocean rings, is a good method to approximate the rings sea surface height structure, but inadequate to represent the dynamical structure of the rings. To obtain $V_{\text{max}}$ accurately, the quadratic model for the inner core must be applied. An approach that can be used with altimetric data is to first use a Gaussian model to estimate $R_{\text{max}}$, and then fit a parabola inside the estimated $R_{\text{max}}$ to obtain refined estimate of $V_{\text{max}}$.

Compared to some previous studies of NBC rings [Goni and Johns, 2003; Fratantoni and Johns, 1995] we find larger (greater $R_{\text{max}}$) and more intense (greater $V_{\text{max}}$) rings. Part of the reason for the lower estimates from altimetry [Goni and Johns, 2003] have to do with inexact crossing of the ring centers during individual satellite passes, and also to the Gaussian model applied to estimate the $V_{\text{max}}$ and $R_{\text{max}}$ parameters. With respect to earlier mooring observations [Fratantoni and Johns, 1995], the difference in the $V_{\text{max}}$ is explained by the greater depth of the velocity measurements used in the previous study.

The independent analysis of two rings using ADCP data and satellite altimetry was in agreement with these predictions. A comparison with the simultaneously sampled in situ rings showed that the altimetric along track analysis tended to underestimate the rings intensity.
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References


**Figure 1.** Circulation schematic of the tropical Atlantic (upper panel). The NBC retroflection is approximated by the dashed line. The shipboard ADCP sampling of rings R–2 and R–3 is illustrated in the lower panel. The star shows the mooring position.

**Figure 2.** Originally surveyed velocity field of ring R–2 (in grey), and the corrected field for a translation speed of 13 cm/s (in black). The best fit ring center is marked by the star. The circle of $1.25R_{\text{max}}$, delimits the subset considered for the center definition.

**Figure 3.** Virtual velocity section of ring M–4, measured from a mooring. The data is positioned in reference to the ring center, marked by the star. The circle, of $1.25R_{\text{max}}$, delimits the subset considered for the center definition. The translation speed, 10 cm/s, was estimated from independent IES measurements, and the center was estimated to pass 30 km away from the mooring.

**Figure 4.** Azimuthal velocity component observed on the 6 radial sections available from R–2. Circles are the observations, the thin lines are the smoothed velocity, the squares are the $R_{\text{max}}$ and $V_{\text{max}}$ for each section and the star is the overall $R_{\text{max}}$ and $V_{\text{max}}$ for the ring.

**Figure 5.** The normalized azimuthal velocity structure of all rings together. The $r^2$ is for the inner ring, and the $\lambda$ is the mean decay scale for the outer ring.

**Figure 6.** Observed azimuthal velocity (dots) for ring R–1 (left) and ring R–2 (right), and the fitted lines for the piecewise model used here (gray), and the CL94 model (solid line).

**Figure 7.** Calculated sea surface slope for all rings sections, normalized by the maximum slope allowed by the gradient balance. The lower panel shows an example of the calculated sea surface slope ($\partial\eta/\partial r$), and theoretical slope limit for a radial section of the ring R-3.
Figure 8. Absolute angular momentum $L$ (upper panel), and on the Rayleigh criterion for inertial instability $\Phi$ (lower panel), for all observed rings.

Figure 9. On the top panel, the black dots are the estimated SSH surface for a radius of $R-2$, and the grey line is a Gaussian fit to that. On the middle panel, the grey line is the respective slope of the Gaussian fit, and the black dash line is the limit for the gradient balance. The lower panel shows the swirl velocity observed in black dots and the estimated velocity from the Gaussian fit applied on the gradient balance. Only the part with a real solution was plotted.

Figure 10. The circles are the SSHA measured by TOPEX and the dashed gray line is its Gaussian fit. The solid line is the section estimated from the shipsurveyed ADCP data. Note that, from the shipsurveyed data, it is shown only the closest section, but its values were determined considering all the available sections for each ring.
Table 1. The mean and standard deviation of derived parameters for the observed rings: $R_{\text{max}}$ (radius of maximum velocity); $V_{\text{max}}$ (maximum swirl velocity); Ro (Rossby number); $\lambda$ (length of outer decay scale); $\eta_0$ (maximum sea surface elevation); $\eta_{\text{in}}$ ($\Delta\eta$ inside the inner ring); $\omega_0$ (inner ring vorticity); $\omega_s$ (“shield” maximum vorticity)

<table>
<thead>
<tr>
<th>Ring</th>
<th>$R_{\text{max}}$</th>
<th>$V_{\text{max}}$</th>
<th>Ro</th>
<th>$\lambda$</th>
<th>$\eta_0$</th>
<th>$\eta_{\text{in}}$</th>
<th>$\omega_0$ $\times 10^{-5}$ s$^{-1}$</th>
<th>$\omega_s$ $\times 10^{-5}$ s$^{-1}$</th>
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<td>R–1</td>
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<td>-0.47</td>
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<td>-1.14±0.21</td>
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<td>38</td>
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<tr>
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